RESEARCH INTERESTS

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I class myself as a mathematical physicist, something I have wanted to be since my undergraduate days. Mathematical physics is to me a very exciting area of research. It not only provides physically motivated problems to which much beautiful abstract mathematics may be applied, but it also repays this debt by providing physical insight and new ideas which have advanced the study of abstract mathematics itself.

I would like to think that my contributions to this field have been characterised by an ability to recognise outstanding problems in mathematical physics, and solve them through the application of original mathematics. One ongoing theme of my work is the quest for precision and transparency in the description of physical models. Whilst this is clearly an artefact of my mathematical training, I believe that it is here that the mathematical physicist can contribute. By emphasising the underlying mathematical structures, one can bring clarity to the murkiest waters of high energy physics.

My research falls under the umbrella of conformal field theory, and I have considerable expertise in the study of rational, quasirational and logarithmic theories. More recently, my attention has been captured by certain classes of non-quasirational theories which are relevant to string theory dualities and integrable models.

1. CURRENT AND FUTURE RESEARCH

My present research at DESY studies integrable deformations of certain classes of conformal field theories. Specifically, we start with free-field realisations of these theories and determine the “hidden” quantum symmetry associated to the deformation in question. The integrable structure of the deformed theory can then be reconstructed using the quantum inverse scattering method, and this can in turn be used to analyse the original conformal field theory. We have thus far only considered toy systems, but these have already turned up surprises, in particular the first example of a quantum symmetry which is not related to an (affine) Lie (super)algebra.

Our aim is to generalise the methods to the point that we can apply them to the problem of determining the spectra of non-linear sigma models with supersymmetry, in particular to the supergroups $PSU(1,1 \mid 2)$ and $PSU(2,2 \mid 4)$. Both these examples are well-known to be relevant to the study of the AdS/CFT correspondence. However, as the target spaces are non-compact, the resulting theories are bound to be hopelessly irrational. The usual methods of conformal field theory therefore cannot be applied.

Moreover, conformal field theories with superalgebra symmetries like these are usually logarithmic, so it is a safe bet that the elucidation of their integrable deformations will require paying close attention to the structures underlying this behaviour. I have therefore complemented my recent research with a study of such logarithmic theories and the analysis of the indecomposable representations which constitute their spectrum. There is some understanding in simple cases, but there remain many questions which have not yet been addressed in the literature. Progress in this area is obviously useful for AdS/CFT, but is also of significant interest in its own right.

A useful model for understanding the behaviour of theory with supersymmetry is to analyse the corresponding behaviour of ordinary bosonic theories at admissible levels. These theories are surprisingly poorly understood, despite their potential role as building blocks for quasirational and logarithmic conformal field theories in general. Furthermore, understanding these theories forms a crucial part of constructing the simplest quasirational theories with superalgebra symmetry. Paradoxically, these are not the integer-level theories (which have received a little attention in the literature), but are instead certain fractional-level versions thereof.

I am also continuing my work on understanding the mathematical foundations of logarithmic conformal field theories in the simplest case of Virasoro symmetry. Here, the standard resource for the structure of the indecomposable representations which arise is an unpublished preprint of Rohsiepe. With Kalle Kytölä, I have reevaluated these results, as I had previously found counterexamples to one of Rohsiepe’s theorems. We have found that the consistent structures are considerably more constrained than Rohsiepe had anticipated, and are currently extending our results to more general physically relevant structures. I also plan to devote some time to a proper discussion of the question of the adjoint in logarithmic conformal field theories. A definition is of course required to compute, but it is not clear that the definitions assumed thus far in the literature are physical, appropriate, or even consistent. This has obvious ramifications for deciding if unitarity can coexist with logarithmic structure.

2. Past Research

My interest in logarithmic conformal field theories with Virasoro symmetry stems from a discussion with Yvan Saint-Aubin on non-local observables in statistical models. With Pierre Mathieu, I then clarified the mathematical structure of the indecomposable representations uncovered in the course of investigating critical percolation as a conformal field theory. This was an unequivocal constructive proof of the fact that critical percolation is described by a logarithmic (boundary) conformal field theory. Our subsequent work then generalised this to other models, noting that the probabilistic models collectively known as stochastic Loewner evolution must therefore also describe indecomposable representations. Our work in percolation is also distinctive in that it stresses the importance of incontrovertible physical input, in our case Cardy's crossing probability formula, to the construction of these conformal field theories. The results have not always been in agreement with those of other groups, and more time needs to be spent reconciling the conflicting interpretations. Partial progress in this direction was reported in my recent paper on Watts' crossing probability, which extended our percolation model by consistently describing a different boundary sector of the theory.

My earlier work with Pierre involved developing the algebraic formulation of conformal field theories whose chiral symmetry algebras are generated by fields exhibiting generalised statistics (that is, are neither bosonic nor fermionic). Whilst such theories have been known for a long time, the representation theory of the associated symmetry algebras has not been significantly developed. The obstruction here is that these algebras are typically defined by so-called generalised commutation relations involving infinitely many terms. In particular, analogues of the Poincaré-Birkhoff-Witt theorem (which gives bases for the Verma modules when the symmetry algebra is a Lie algebra) have not been established for these algebras, even though this is fundamental to the connection with fermionic character formulae and combinatorics. Our work has made some inroads into solving this difficult problem, which is relevant to models exhibiting (abelian) anyonic statistics. The hallmark example being of course the fractional quantum hall effect.

One of the interesting realisations of our work is that many of the best-known rational conformal field theories may be recast as theories whose symmetry algebra acts faithfully (that is, the modules comprising the theory have no singular vectors). This therefore gives a natural "free-field representation" of these theories for which no BRST projection is necessary. The trade-off for this recasting is that the symmetry algebra is defined by generalised commutation relations. Our work here has already clarified (and simplified) many previous results. I have also recently shown that our formalism also applies to certain quasirational conformal field theories, and forsee no difficulties with logarithmic theories as well.

My doctoral research with Peter Bouwknegt concentrated on properties of certain conformal field theories called Wess-Zumino-Witten models. These are string theories defined on the underlying manifold of a suitable Lie group, and provide the most tractable examples of string theories on curved space. My thesis concerned certain dynamical objects (D-branes) which appear in string theories, and investigated the conserved charges that are associated to these objects and the abelian groups that these charges take values in. I played an active role in generalising the computation of these charge groups from theories over the special unitary groups to theories over the other simple Lie groups. I also subsequently developed a framework in which these conserved charges could be derived and studied rigorously using the global topology and differential geometry of the D-branes themselves, rather than by low-energy field-theoretic approximations and renormalisation group flows. This framework allowed me to explain several intriguing observations made earlier, and more importantly, proved that these D-brane charges and charge groups coincided with those expected from geometric principles. I also contributed a thorough discussion of the (then) current knowledge of the fusion rings associated to these models, and proved several new results in this area. I would be very interested in revisiting this field, especially brane charges, in the context of string theories defined on Lie supergroups.

The fields of physics and mathematics that I am currently interested in therefore include the foundations of conformal field theory (and vertex operator algebras), the representation theory of infinite-dimensional Lie algebras and superalgebras (especially the indecomposable aspects), their relation to integrability (quasitriangular Hopf algebras), and stochastic Loewner evolution. I also work, or have worked, with aspects of string theory, compact Lie group theory, cohomology of group manifolds and their quotients, commutative algebra, and homological algebra describing categories of Lie algebra modules.