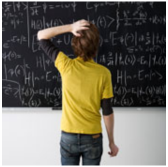
Anything you can do...



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Founder's Day,
October 15, 2010

The Tao of TP

Theoretical physicists build mathematical models to (try to) explain natural phenomena.

Mathematical physicists concern themselves with the mathematics underlying these endeavours.

At the ANU, we study both. More specifically, we study:

- Exactly Solvable Lattice Models
- Condensed Matter Physics
- Non-linear Optics
- Conformal Field Theory
- Stringy Geometry

and all the fun math that we need for this.

We avoid laboratories. A true theoretical physicist never includes pictures of labs in their seminars.

THIS NEVER HAPPENS!!!

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OUR WORKSPACES AREN'T PRETTY!!!

Physicists and their Toys

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- They're easier to study than the real world.
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A perfect example is the theory of phase transitions:

First Order

Second Order

Second order phase transitions are observed in many toy models, *eg*. percolation and the Ising model.

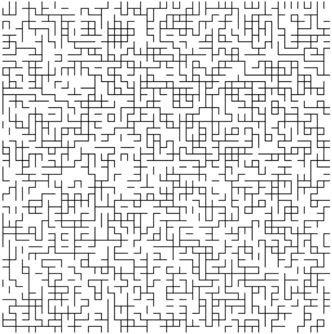
At the critical point, the limiting theory is conformally invariant, hence physicists use conformal field theory in computations.

This has been extremely successful.

Example: Percolation

Percolation is one of the simplest lattice models.

A central question is:
Given that each bond is open with probability p, what is the probability π that a random configuration of bonds will admit an open path from the west wall to the east wall?



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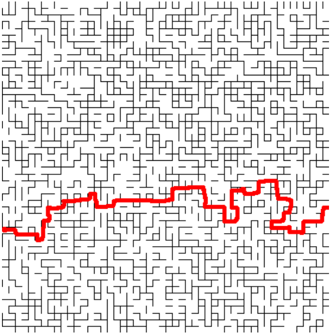

In the thermodynamic limit,

$$\pi = \begin{cases} 0 & \text{if } p < p_c, \\ 1 & \text{if } p > p_c, \end{cases}$$

where p_c is the critical probability. This is therefore a simple model of a phase transition.

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It remains to calculate π at $p = p_c$ as a function of the aspect ratio of the domain.

This can be done numerically with a computer. However, in one of the most celebrated applications of conformal field theory, John Cardy (1991) provided a closed-form expression for the function π in the thermodynamic limit.

Enter the Mathematicians

At the time, the mathematical community were none too happy. As Cardy (2005) put it, referring to his percolation result:

With this result, the simmering unease that mathematicians felt about these methods came to the surface. What exactly are these renormalised local operators whose correlation functions the field theorists so happily manipulate, according to rules that sometimes seem to be a matter of cultural convention rather than any rigorous logic? What does conformal symmetry really mean? Exactly which object is conformally invariant?

Aside from these deep concerns, there was perhaps also the territorial feeling that percolation theory, in particular, is a branch of probability theory, and should be understood from that point of view, not merely as a by-product of quantum field theory.

Luckily for them, Oded Schramm soon (2000) introduced stochastic Loewner evolution.

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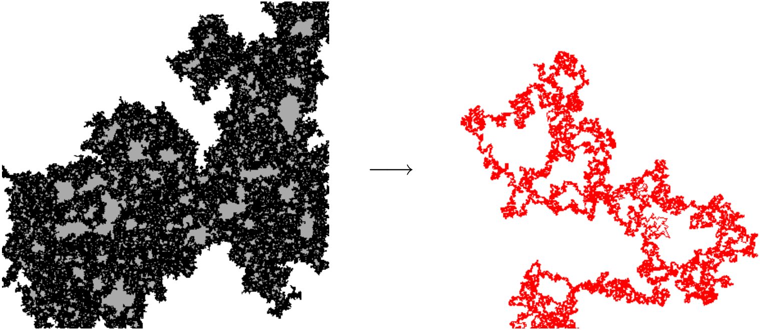
Luckily for them, Oded Schramm soon (2000) introduced Schramm Loewner evolution. This is a probabilistic description of fractal curves in the continuum.



SLE describes the limits of cluster boundaries. For percolation, the fractal dimension is $\frac{7}{4}$.

Schramm's collaborator, Wendelin Werner received a Fields medal in 2006 for helping develop SLE.

Stas Smirnov received a Fields medal in 2010 for proving that (a variant) of percolation converges to a certain SLE.

Schramm

Werner

Smirnov

Suddenly, there are groups of mathematicians industriously proving things known to physicists (and more!).

Logarithmic CFT

Many things that SLE enthusiasts study, such as fractal dimensions and crossing probabilities, do not fit naturally within the framework of CFT.

Recently, physicists have realised that these non-local observables are more naturally accommodated by a generalisation, known as logarithmic CFT.

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LCFT was originally introduced by Rozansky and Saleur (1991) and Gurarie (1993), but largely ignored. The first ever LCFT conference was only held in May, 2009.

At the ANU (and elsewhere), work is underway to:

- Explore fundamental examples of LCFTs and how they relate to one another,
- Understand the mathematical structures that distinguish LCFT from CFT,
- Develop computational methods for applications of LCFTs,
- Research and characterise the LCFT/SLE correspondence.

In this way, physicists are rising to the challenge. LCFT research will not only further our knowledge of models like percolation, but will also contribute to string theory and pure mathematics research.