

**Multi-level model analysis of the
Knowledge and Skills scale
of the NAEP 1986 math data
(final report)**

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Contents

1	Aim of the project	4
2	Summary	4
3	Data and model specification	5
3.1	Two-level models	5
3.2	Three-level models	7
3.3	Four-level models	7
4	Analysis	8
4.1	2- and 3-level models	8
4.1.1	Log-likelihoods	8
4.1.2	Variance components	9
4.1.3	Parameter estimates	9
4.1.4	Standard errors	9
4.2	4-level models	9
4.2.1	Log-likelihoods	9
4.2.2	Variance components	10
4.2.3	Parameter estimates	10
4.2.4	Standard errors	10
5	Computational issues	10
6	Presentation of parameter estimates and SEs	11
7	Relation of modeling results to unpublished ETS tables	11
8	Reporting group differences from modeling and from the unpublished 1986 tables	13
9	Summary of reporting group parameters from the model	14
10	Item parameter estimates	14
11	Comparison of item parameters estimated from ETS and ML	15
12	Acknowledgements	16
13	References	16
14	Appendix 1 – model parameter estimates and SEs	17
15	Appendix 2 – item parameter estimates	24

16	Appendix 3 – ETS and ML item estimates and SEs	27
17	Appendix 4 – parameter correlation matrix for 3-level Rasch model	28
18	Appendix 5 – parameter correlation matrix for 4-level 2PL model	36

1 Aim of the project

The aim of this project was to assess the feasibility of analyzing NAEP-scale data using multi-level models to represent the clustering of the NAEP sample design at all four levels: PSUs, schools, students and items, and to compare the results where possible with reported results from 1986.

2 Summary

The study established that the multi-level model analysis of NAEP-scale data allows properly for the survey design, gives efficient and correct standard errors, and is computationally feasible. Computing times with the Gllamm package in Stata were substantial for the three-level models, and very substantial for the four-level models: efficient algorithms (for example, a parallelized form of the EM algorithm) are essential for the routine multi-level analysis of NAEP-scale data.

In more detail:

- the 2PL models had higher maximized log-likelihoods than the corresponding MIMIC models at all levels, and both had much higher log-likelihoods than the Rasch model: the four-level 2PL model provided the best representation of the test data, with the three-level 2PL model very little worse;
- the two-level models (ignoring the PSU- and school-level clustering) gave seriously biased estimates of the black-white difference, and underestimated standard errors for all the reporting group parameters except sex and ethnic group;
- the three-level models were sufficient to give correctly specified estimates and standard errors for the reporting group variables except for the standard errors for the region and size and type of community parameters which were slightly underestimated;
- the fourth level had a very small (though non-zero) variance component, which had only slight effects on the estimates and standard errors of the parameters at the upper levels, relative to those from the three-level model;
- the standard errors of the reporting group parameter estimates were substantially larger than those from the (unpublished) 1986 reporting group comparisons;
- the item intercept parameter estimates in all the models at all levels were extremely highly inter-correlated; this is one of the main computational difficulties in maximum likelihood for these models.

3 Data and model specification

There were 21,287 Grade 3/Age 9 students in the survey, but only about half of these had responses on any of the items on the Knowledge and Skills scale, so the “full” data set for this scale has 10,463 students clustered in 440 schools, which are themselves clustered in 94 PSUs. High-minority schools were over-sampled to ensure adequate minority student samples. This over-sampling does not require weighting in the analysis as both the school identifier and the student ethnicity are retained in all model analyses except those using the two-level model, in which the school is not identified.

The number of students per school varied from 5 to about 45, with an average of 24, and there was an average of 7 items answered per student. The item responses were coded 0 or 1 according to the manual, with items skipped coded zero and items not reached treated as missing and omitted from the data set.

We used a minimal set of reporting variables: sex, race (6 levels), region (4), size and type of community (stoc, 7) and parents education level (pared, 6), to give us some feel for the results. We used a main effect model with 20 dummy variables for these categorical variables.

The data CD had only the scrambled PSU identifier, which is essentially a school identifier - we could identify only a three-level model of schools, students and items. We later received the unscrambled code from ETS which allowed the 4-level analyses.

In analyzing the test data, we evaluated the three main item response models: the Rasch, 2PL and MIMIC models. The Rasch is not generally used for NCEES test items but is of historical interest and provides a base for assessing the value of the discrimination parameters in the two-parameter models. The 2PL and MIMIC models have different forms for the regression and in an earlier report we noted that, although the models have the same number of parameters, they can be distinguished with sufficient data.

3.1 Two-level models

For student i with latent ability z_i (assumed one-dimensional) on the items of the test scale, and covariates (reporting group variables) \mathbf{x}_i , attempting item j and giving the binary responses y_{ij} with probabilities p_{ij} for a correct answer, the Rasch model has the form

$$\begin{aligned} \text{logit } p_{ij} \mid z_i &= \alpha_j + z_i \\ z_i &\sim N(\boldsymbol{\gamma}'\mathbf{x}_i, \sigma^2) \end{aligned}$$

where $\boldsymbol{\gamma}$ is the vector of regression coefficients – the *reporting group parameters*. In this formulation the latent ability is regressed on the explanatory variables, but we can transfer the regression model to the logit scale, by

defining

$$\begin{aligned} z'_i &= z_i - \boldsymbol{\gamma}'\mathbf{x}_i, \\ z_i &= z'_i + \boldsymbol{\gamma}'\mathbf{x}_i \end{aligned}$$

and then dropping the prime from z'_i , to obtain the Rasch model in the form

$$\begin{aligned} \text{logit } p_{ij} \mid z_i &= \alpha_j + z_i + \boldsymbol{\gamma}'\mathbf{x}_i \\ z_i &\sim N(0, \sigma^2). \end{aligned}$$

Now ability has a homogeneous distribution, and the explanatory variables, through the regression model, affect directly the (logit of the) probability of a correct answer. These two formulations of the Rasch model are equivalent and indistinguishable.

The 2PL model has the form

$$\begin{aligned} \text{logit } p_{ij} \mid z_i &= \alpha_j + \beta_j z_i + \boldsymbol{\gamma}'\mathbf{x}_i \\ z_i &\sim N(0, \sigma^2), \end{aligned}$$

and the MIMIC model has the form

$$\begin{aligned} \text{logit } p_{ij} \mid z_i &= \alpha_j + \beta_j z_i \\ z_i &\sim N(\boldsymbol{\gamma}'\mathbf{x}_i, \sigma^2), \end{aligned}$$

or, by transferring the regression model to the logit scale as above:

$$\begin{aligned} \text{logit } p_{ij} \mid z_i &= \alpha_j + \beta_j z_i + \beta_j \boldsymbol{\gamma}'\mathbf{x}_i \\ z_i &\sim N(0, \sigma^2). \end{aligned}$$

The MIMIC model has the regression model with the reporting group variables “inside” the student ability distribution; the 2PL model has it “outside”, in the logit model for the item responses. In the Rasch and 2PL models the effects of the reporting group variables and individual items are *additive* on the logit scale, whereas in the MIMIC model item slopes *interact* with the reporting group variables on the logit scale, so that the effects of the reporting group variables on the item response probabilities are scaled by the item discriminations and so are *different* for each item.

The first form of the Rasch and MIMIC models has the strong property that, given ability, the item response probabilities *do not depend on the explanatory variables*. The second form however does *not* have this property – the explanatory variables now appear explicitly in the logit of the item response probabilities. Since the two forms of the models are interchangeable, this makes clear that the usual definition of item bias – that, given ability, the item response probabilities are different for different reporting groups – is not a satisfactory definition, and that some form of *interaction* definition is needed.

The MIMIC model (the name comes from structural equation models : Multiple Indicators and Multiple Causes of a single latent variable) is the standard model currently used in the analysis of NAEP data – it is usually called the “2PL model” in the IRT literature. Skrondal and Rabe-Hesketh (2004) discuss these models in detail.

The 2PL and MIMIC models are identical when the regression model is null, that is when $\boldsymbol{\gamma} = \mathbf{0}$, but are in general different, though they have the same number of parameters. The two models can be discriminated with sufficient data.

3.2 Three-level models

The three-level models include an additional random effect η_k for the effect of school k . We give the models in the alternative form with a zero mean for the ability random effect z_i . For the Rasch model,

$$\begin{aligned}\text{logit } p_{ijk} \mid z_i, \eta_k &= \alpha_j + z_i + \boldsymbol{\gamma}' \mathbf{x}_i + \eta_k \\ z_i &\sim N(0, \sigma^2) \\ \eta_k &\sim N(0, \sigma_{sch}^2),\end{aligned}$$

for the 2PL model

$$\begin{aligned}\text{logit } p_{ij} \mid z_i, \eta_k &= \alpha_j + \beta_j z_i + \boldsymbol{\gamma}' \mathbf{x}_i + \eta_k \\ z_i &\sim N(0, \sigma^2) \\ \eta_k &\sim N(0, \sigma_{sch}^2)\end{aligned}$$

and for the MIMIC model,

$$\begin{aligned}\text{logit } p_{ijk} \mid z_i, \eta_k &= \alpha_j + \beta_j z_i + \beta_j \boldsymbol{\gamma}' \mathbf{x}_i + \beta_j \eta_k \\ z_i &\sim N(0, \sigma^2) \\ \eta_k &\sim N(0, \sigma_{sch}^2).\end{aligned}$$

3.3 Four-level models

The four-level model adds a PSU-level random effect ϵ_ℓ for the effect of PSU ℓ ; the notation for the school effect changes to $\eta_{k\ell}$ for the effect of school k in PSU ℓ , and the “among schools” variance component for schools in the three-level model is decomposed into an “among PSU” component and an “among school within PSU” component.

For the Rasch model,

$$\begin{aligned}\text{logit } p_{ijkl} \mid z_i, \eta_{k\ell}, \epsilon_\ell &= \alpha_j + z_i + \boldsymbol{\gamma}' \mathbf{x}_i + \eta_{k\ell} + \epsilon_\ell \\ z_i &\sim N(0, \sigma^2) \\ \eta_{k\ell} &\sim N(0, \sigma_{sch}^2) \\ \epsilon_\ell &\sim N(0, \sigma_{PSU}^2);\end{aligned}$$

for the 2PL model

$$\begin{aligned} \text{logit } p_{ijkl} \mid z_i, \eta_{kl}, \epsilon_\ell &= \alpha_j + \beta_j z_i + \boldsymbol{\gamma}' \mathbf{x}_i + \eta_{kl} + \epsilon_\ell \\ z_i &\sim N(0, \sigma^2) \\ \eta_k &\sim N(0, \sigma_{sch}^2) \\ \epsilon_\ell &\sim N(0, \sigma_{PSU}^2), \end{aligned}$$

and for the MIMIC model,

$$\begin{aligned} \text{logit } p_{ijkl} \mid z_i, \eta_{kl}, \epsilon_\ell &= \alpha_j + \beta_j z_i + \beta_j \boldsymbol{\gamma}' \mathbf{x}_i + \beta_j \eta_{kl} + \beta_j \epsilon_\ell \\ z_i &\sim N(0, \sigma^2) \\ \eta_{kl} &\sim N(0, \sigma_{sch}^2) \\ \epsilon_\ell &\sim N(0, \sigma_{PSU}^2). \end{aligned}$$

The three- and four-level MIMIC models have the same property as in the two-level model: the additional terms in the ability model interact with the item slopes to give differently scaled random effects on the logit scale for each item.

4 Analysis

We began the analysis with 2-level Rasch, 2PL and MIMIC models. We ran all 3-level Rasch, 2PL and MIMIC models using the parameter estimates from the corresponding 2-level models as starting values. The 3-level parameter estimates were used as starting values for the corresponding 4-level models.

Null models (items but no reporting group variables) were also fitted to establish the size of the variance components and the importance of the regression.

4.1 2- and 3-level models

4.1.1 Log-likelihoods

The log-likelihood improvement for the 3-level model over the 2-level model is large in every case – 126 for the Rasch, 147 for the 2PL and 118 for the MIMIC. A formal test (the “chi-bar” test) treats the deviance difference – twice the log-likelihood difference – as a mixture of χ^2 distribution: $0.5\chi_0^2 + 0.5\chi_1^2$. The first component is a degenerate distribution – a discrete “spike” mass of 0.5 at zero: if the null hypothesis is true, the MLE of the variance component is zero about half the time, and so the maximized likelihood under the alternative hypothesis will be the same as that under the null half the time, and the other half it will behave like χ_1^2 . The deviance difference

vastly exceeds the critical value in all three models. The 2-level model does not provide a correct representation of the clustered survey design.

The 2-level 2PL model fitted the test data somewhat better than the 2-level MIMIC model – a log-likelihood difference of 3.06 – and vastly better than the Rasch model – a log-likelihood difference of 397.96.

For the 3-level model the differences in favour of the 2PL are much greater – 31.63 compared with the MIMIC model and 418.99 with the Rasch model.

4.1.2 Variance components

The school variance components at the third level were quite large, relative to the student level variances. These were respectively 0.154 and 1.168 for the Rasch, 0.139 and 1.682 for the 2PL, and 0.135 and 1.065 for the MIMIC.

4.1.3 Parameter estimates

The parameter estimates changed substantially at the student level from the 2-level to the 3-level model – the black (compared with white) estimate reduced by 2.5 standard errors for the Rasch model, 2.7 SEs for the 2PL model and 1.6 SEs for the MIMIC model, and the American Indian - white difference reduced by one SE for all three models. The other parameters were more stable, the largest change being one SE for the lomet category of stoc (relative to extreme rural).

The very serious bias of the black-white difference in the 2-level model shows the importance of modeling the design correctly with the school level random effect.

4.1.4 Standard errors

The intra-class (school) correlations are 0.117 for the (3-level) Rasch, .076 for the 2PL and .113 for the MIMIC. These are not large values but the corresponding design effects change the standard errors quite substantially – these are increased by 50% relative to those from the 2-level model for variables at the school level (like region and stoc). The standard errors for parents education are also increased by the same proportion, although this is apparently a student-level variable. This probably reflects the homogeneity of educational level amongst parents of children in the same school. The standard errors of the student-level demographic variables are little affected.

4.2 4-level models

4.2.1 Log-likelihoods

The log-likelihood improvement for the 4-level model over the 3-level model is much smaller than that for the 3- to 2-level model in every case – 6.19

for the Rasch, 5.45 for the 2PL and 7.22 for the MIMIC. The difference is statistically significant at the 0.1% level for all three.

The 2PL model again fitted much better than the MIMIC – a log-likelihood improvement of 29.86 – and vastly better than the Rasch – 418.25. This strongly suggests (although there is no formal Neyman-Pearson test for this model comparison) that the interaction terms on the logit scale in the MIMIC model are unnecessary, and the simpler structure of the 2PL model is preferable to the MIMIC model.

4.2.2 Variance components

The PSU variance components in all three cases, though significant, were very small – 0.036 for the Rasch, 0.019 for the 2PL and 0.032 for the MIMIC.

The small change in log-likelihood, and the very small variance components, suggest that the 3-level model provides a substantially correct representation of the clustered survey design: the variation among PSUs was very small after the variation among schools within PSUs was taken out.

4.2.3 Parameter estimates

The effect of the fourth level estimation on the model parameter estimates was small, with the largest change being 0.33 SE in one stoc parameter. The parents education estimates all increased (relative to the three-level model), by 0.05 for the Rasch model, 0.02 for the 2PL model and 0.04 for the MIMIC. These changes represent 25% of a standard error for the Rasch model, 10% for the 2PL model and 20% for the MIMIC. The gender and ethnic origin parameters and standard errors were very stable from three to four levels.

4.2.4 Standard errors

The effect on the standard errors was also small – the SEs of the region effects increased by about 20%, while those of the parents education *decreased* by about 5%, because the school variance component was reduced by the fourth level modeling.

5 Computational issues

Computational times for these models in Gllamm were very substantial. This is partly due to the near-singularity of the information matrix in the more complex models; a singular information matrix in a regression model results from linear dependence in the explanatory variables, and this prevents the inversion of the matrix and the estimation of parameters and their standard errors. The severity of the near-singularity is usually measured by

the *condition number* which is reported by Glamm. The condition number is the ratio of the largest to the smallest eigenvalue of the information matrix – the larger the condition number, the closer is the information matrix to singularity.

All the models were badly conditioned except for the Rasch null models. The table below gives the condition number for most of the models: the MIMIC model was consistently the worst-conditioned.

	Condition numbers		
	-----	-----	-----
	Rasch	2PL	MIMIC
	-----	-----	-----
2-level null	6.0	42.3	42.3
3-level null	11.0	36.7	
4-level null	12.7	37.1	
2-level full	63.0	70.5	75.4
3-level full	83.3	67.9	106.4
4-level full	83.0	83.2	101.4

6 Presentation of parameter estimates and SEs

Results for all models are given in Appendix 1. The reporting group parameter estimates are given on the $N(0,1)$ latent ability scale (for the Rasch and MIMIC models) or the logit scale (for the Rasch and 2PL models). For the Rasch model the scales are identical.

7 Relation of modeling results to unpublished ETS tables

The ETS tables relevant to our results are the one-way tabulations of plausible values by (total, Gender, Race/ethnicity, School Type, Parental Education, Region). Our model used Size and Type of Community (stoc), and not School Type.

The one-way tabulations will in general give different results from the main effect model estimates since any correlation between the reporting group variables will affect the marginal mean differences. (This is the reason for fitting all reporting groups together in a main effect regression model.)

To compare the results, we present the ETS tables in the same form as the MIMIC model parameters, with the reference category mean subtracted, and standard errors calculated from the variances of the difference between the means. We rescale the MIMIC parameters to the NAEP reporting scale with an origin of 250.5 and standard deviation 50, for the reference category of the dummy variables (male, white, region 1, stoc 1, pred 1).

Variable	ETS table results		MIMIC model estimates	
	Estimate	SE	MLE	SE
Male	0		0	
Female	2.2	1.5	6.2	1.6
White	0		0	
Black	-31.1	2.1	-32.6	4.1
Hispanic	-22.4	2.4	-23.2	3.2
Asian/Pacific Islander	- 3.5	5.9	- 8.3	6.3
American Indian	-13.0	3.4	-24.3	5.8
other	5.5	16.9	8.3	35.8
NorthEast	0		0	
SouthEast	- 2.4	2.1	- 3.8	3.9
Central	- 1.3	2.3	- 9.8	3.8
West	- 4.6	2.5	- 9.7	3.5
ExtremeRural			0	
Lo Metrop			-16.0	6.3
Hi Metrop			22.3	6.6
MainCity			6.5	6.0
UrbanFringe			8.5	6.2
MediumCity			4.9	5.7
SmallPlace			- 2.2	5.4
NotFinHiSch	0		0	
FinHiSch	7.8	4.0	- 1.0	10.9
SomeColl	22.8	4.3	8.4	10.6
CollGrad	24.9	3.8	28.5	11.2
DontKnow	13.1	3.7	27.7	10.9
NoResponse	- 5.2	*	10.4	10.5

* SE not given

8 Reporting group differences from modeling and from the unpublished 1986 tables

Some reporting group differences are quite similar in the two approaches. The Black-White and Hispanic-White group differences based on large samples agree closely, and those based on small samples do not differ by more than one standard error except the American Indian group. However the other variables do not show good agreement, and there are some large differences in parental education groups, with the ETS tables giving equivalent means for “some college” and “college graduate” levels, while the ML analysis shows college graduates 20 points above the “some college” group, and much closer to the “don’t know” group.

These differences may be due partly to the substantial differences in the item parameter estimates in the two analyses, discussed further below. Another possibility, as noted above, may be the difference between marginal tabulations (by each of two variables) and the two-way tabulation by the same two variables, or by a full main-effect model-based analysis, depending on the correlation between the tabulating variables. This raises an important issue of how these differences should be assessed and reported; the occurrence of Simpson’s paradox in such marginal tabulations is well-documented and always important to assess.

Of greater concern are the very substantial differences in standard errors. Those reported in the ETS tables, apart from sex, are around half those found from the 3-level ML analysis; the ETS standard errors are much closer to those from the 2-level model analysis ignoring the PSU and school clustering. The reason for this is unclear, but it is certainly not that the 3-level ML analysis ignores the clustering: the intra-class correlation is estimated at 0.11, and this automatically adjusts the standard errors for the cluster design. The clustering affects mainly the upper-level variables in the model: standard errors of the upper-level variables are nearly 50% larger in the 3-level analysis than in the 2-level analysis.

Nor is the reason the use of the full reporting group model in the ML analysis: if a set of explanatory variables are highly correlated, the standard errors for the parameter estimates from the full regression model may be much larger than those from the regressions with each single variable. There are certainly some correlations among the ethnicity, type of community and parents education variables, and this possibility can be assessed by examining the correlation matrix of the parameter estimates. This is given for the 3-level Rasch model in Appendix 4 and the four-level 2PL model in Appendix 5. The correlations of the reporting group parameter estimates are very low, apart from the positive inter-correlations for the parameters for each category of a single variable. So the fitting of the full reporting group model does not induce the large standard errors for this model.

Another possibility is the approximation used to compute sampling variances of subgroup means used in the NAEP technical report and in the ETS tables. The differences in standard errors warrant further investigation.

9 Summary of reporting group parameters from the model

The parameter estimates from the above table are interpreted here for the MIMIC model:

- Girls have a slightly higher mean than boys;
- Blacks, Hispanics and American Indians have much lower means than Whites, Asian/ Pacific Islanders are somewhat below Whites, though not significantly, and “others” are similar to Whites;
- the SouthEast region is similar to NorthEast while West and Center are lower and similar;
- Low Metropolitan is substantially lower than Extreme Rural; High Metropolitan is much higher; Main City, Urban Fringe, Medium City and Small Place are similar to Extreme Rural;
- Students whose parents had less than a full college degree have similar means, and similar to the No Response group. The Don’t Know group are similar to the College Graduate group and have much higher means than the other groups.

10 Item parameter estimates

We give in Appendix 2 the item parameters from the three sets of models. A notable feature of the Rasch item parameter estimates is their *invariance* across the different models – the 2-level, 3-level and 4-level models, whether full or null, have essentially the same parameter estimates. Their standard errors however are increased substantially in the full 2-level model compared to the null model (because of the correlations between parameter estimates discussed below), and increase further in the full 3-level model (because of the substantial school design effect), but do not increase further in the 4-level model (because of the very small PSU design effect).

The Rasch item parameter estimates from different models differ only by an origin term, and are given in the appendix with an origin shift so that they all have the same average difficulty. This strong invariance led us to investigate the correlation matrix of the item parameters and reporting group parameters. In an earlier draft of this report we conjectured that the

Rasch model information matrix (and therefore the correlation matrix of the estimated parameters) was block-diagonal. This matrix is available as part of the program output, and is shown (to 3 dp) for the Rasch model in Appendix 4.

There is a strikingly high intercorrelation of the item intercepts, averaging nearly 0.95, with much lower correlations between item parameters and reporting group parameters, except for the parents education parameters which are correlated around -0.8 with the item parameters. However the matrix is *not* block-diagonal, and the reason for the stability of the Rasch item estimates is probably the existence of sufficient statistics for both item parameters and reporting group parameters.

The very high inter-correlations of the item parameter estimates explains the high condition number, and the difficulty in maximizing the likelihood for both the Rasch model and the much more complex MIMIC and 2PL models; we show the correlation matrix for the 4-level 2PL model in Appendix 5. The item intercepts show a very similar pattern to those in the Rasch model, but the slopes are almost uncorrelated with both the intercepts and the reporting group parameters; the slopes are themselves moderately intercorrelated, with an average correlation of about 0.6.

The 4-level MIMIC model had a non-positive definite information matrix, and the correlation matrix of the estimated parameters could not be computed because of zero or negative values of the variances on the main diagonal of the parameter covariance matrix. This result and the higher log-likelihood of the 2PL model suggest that the 2PL model is preferable to the MIMIC model for these data.

11 Comparison of item parameters estimated from ETS and ML

The item parameters a_j, b_j and c_j reported by ETS are shown in Appendix 3, with the 3-level MIMIC estimates from the present analysis rescaled to correspond to the ETS coding: since

$$a_j(z - b_j) = \alpha_j + \beta_j z,$$

we have

$$a_j = \beta_j, b_j = -\alpha_j/\beta_j.$$

Some of the items use the 3PL model with the c_j guessing parameter, and so are not comparable with the two-parameter models used in our analysis. The two sets of estimates differ very considerably; this is due partly to the difference between the “separate” and “joint” estimation methods used in 1986 and the ML analysis, partly to our adjustment for the clustered survey design, and partly to different codings used for “intentional omissions”. In

our analysis, following the manual, we coded these as incorrect (0), whereas the User Guide 1986 states that these were coded as “partially correct”, with value the reciprocal of the number of alternatives.

The difference in standard errors for the a parameters is even greater: the median SE for the ETS a estimates is .025, that for the ML estimates is .134, more than five times as large. The reason for this is unclear – the ETS estimates were produced by LOGIST on the same data. The different coding for intentional omissions is unlikely to be responsible.

12 Acknowledgements

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13 References

Skrondal, A. and Rabe-Hesketh, S. (2004) *Generalized Latent Variable Modeling: Multilevel, Longitudinal and Structural Relations Models*. Chapman and Hall/CRC, Boca Raton, FL.

14 Appendix 1 – model parameter estimates and SEs

Rasch variance component estimates and SEs for the null models

	2-level	3-level	4-level
s^2_{PSU}	0	0	.059(.016)
s^2_{sch}	0	.325(.029)	.257(.024)
s^2	1.558(.043)	1.246(.038)	1.243(.038)
log L	-40,977.29	-40,605.49	-40,601.08

2PL variance component estimates and SEs for the null models

	2-level	3-level	4-level
s^2_{PSU}	0	0	.033(.013)
s^2_{sch}	0	.285(.026)	.212(.019)
s^2	1.574(.320)	1.665(.358)	1.634(.350)
log L	-40,559.10	-40,183.23	-40,180.08

Rasch, MIMIC and 2PL estimates and SEs for the 2-level model

	Rasch	MIMIC	2PL
male	0	0	0
femal	.111(.031)	.121(.033)	.015(.028)
white	0	0	0
black	-.830(.045)	-.780(.090)	-.792(.041)
hispa	-.546(.045)	-.526(.069)	-.514(.041)
as/pa	-.214(.126)	-.188(.117)	-.240(.114)
amind	-.599(.113)	-.619(.121)	-.588(.092)
other	-.121(.783)	.230(.672)	-.380(.855)
NE	0	0	0
SE	-.033(.051)	-.045(.049)	.003(.046)
Cent	-.245(.051)	-.230(.055)	-.219(.046)
West	-.209(.046)	-.213(.049)	-.193(.042)
extru	0	0	0
lomet	-.224(.083)	-.207(.085)	-.126(.075)
himet	.508(.081)	.505(.094)	.475(.074)
manct	.141(.079)	.174(.079)	.133(.072)
urbfr	.161(.081)	.173(.082)	.121(.074)
medct	.079(.074)	.079(.073)	.031(.067)
smp1c	-.066(.072)	-.036(.072)	-.055(.066)
nfnhs	0	0	0
finhs	-.105(.159)	-.086(.154)	-.205(.152)
smcol	.120(.146)	.145(.140)	.061(.141)
colgr	.593(.154)	.595(.160)	.460(.149)
DK	.591(.142)	.594(.149)	.453(.138)
nores	.165(.141)	.194(.136)	.047(.137)
s ²	1.311(.039)	1.185(.244)	1.646(.345)
logL	-40,475.22	-40,080.32	-40,077.26

Rasch, MIMIC and 2PL estimates and SEs for the 3-level model

	Rasch	MIMIC	2PL
male	0	0	0
femal	.107(.031)	.124(.032)	.012(.028)
white	0	0	0
black	-.703(.050)	-.652(.082)	-.667(.047)
hispa	-.492(.047)	-.464(.065)	-.460(.043)
as/pa	-.196(.129)	-.167(.126)	-.203(.117)
amind	-.489(.106)	-.487(.117)	-.471(.093)
other	-.054(.718)	.167(.717)	-.200(.752)
NE	0	0	0
SE	-.056(.081)	-.077(.078)	-.020(.077)
Cent	-.204(.080)	-.196(.077)	-.172(.074)
West	-.204(.074)	-.194(.071)	-.182(.069)
extru	0	0	0
lomet	-.310(.127)	-.319(.127)	-.201(.113)
himet	.511(.125)	.447(.133)	.497(.116)
manct	.143(.119)	.131(.120)	.150(.106)
urbfr	.188(.125)	.171(.125)	.158(.112)
medct	.113(.112)	.098(.114)	.092(.097)
smp1c	-.038(.108)	-.045(.109)	-.019(.095)
nfnhs	0	0	0
finhs	-.033(.216)	-.020(.219)	-.179(.206)
smcol	.186(.208)	.168(.213)	.045(.200)
colgr	.607(.214)	.570(.225)	.398(.205)
DK	.588(.206)	.554(.218)	.382(.198)
nores	.221(.205)	.209(.211)	.027(.197)
s^2_sch	.154(.019)	.135(.032)	.139(.017)
s^2	1.168(.037)	1.065(.219)	1.682(.365)
logL	-40,349.04	-39,961.68	-39,930.05

Rasch, MIMIC and 2PL estimates and SEs for the 4-level model

	Rasch	MIMIC	2PL
male	0	0	0
femal	.107(.031)	.124(.032)	.012(.028)
white	0	0	0
black	-.707(.050)	-.657(.082)	-.670(.046)
hispa	-.491(.047)	-.462(.065)	-.454(.043)
as/pa	-.199(.129)	-.176(.127)	-.206(.116)
amind	-.493(.105)	-.494(.115)	-.471(.093)
other	-.043(.717)	.198(.721)	-.174(.739)
NE	0	0	0
SE	-.050(.095)	-.071(.088)	-.036(.080)
Cent	-.203(.098)	-.180(.096)	-.176(.087)
West	-.184(.089)	-.184(.083)	-.198(.077)
extru	0	0	0
lomet	-.300(.126)	-.332(.123)	-.189(.120)
himet	.471(.127)	.408(.123)	.481(.119)
manct	.137(.123)	.134(.114)	.188(.121)
urbfr	.179(.125)	.167(.117)	.176(.122)
medct	.069(.103)	.069(.108)	.061(.110)
smplc	-.052(.107)	-.064(.100)	-.026(.102)
nfnhs	0	0	0
finhs	.017(.213)	.018(.211)	-.153(.201)
smcol	.234(.205)	.203(.205)	.065(.194)
colgr	.655(.211)	.608(.219)	.422(.199)
DK	.636(.202)	.592(.211)	.403(.192)
nores	.270(.202)	.245(.203)	.048(.191)
s ² _PSU	.036(.013)	.032(.014)	.019(.010)
s ² _sch	.116(.019)	.108(.028)	.119(.017)
s ²	1.169(.037)	1.078(.221)	1.653(.360)
logL	-40,342.85	-39,954.46	-39,924.60

Rasch estimates and SEs for the 2-, 3- and 4-level models

	2-level	3-level	4-level
male	0	0	0
femal	.111(.031)	.107(.031)	.107(.031)
white	0	0	0
black	-.830(.045)	-.703(.050)	-.707(.050)
hispa	-.546(.045)	-.492(.047)	-.491(.047)
as/pa	-.214(.126)	-.196(.129)	-.199(.129)
amind	-.599(.113)	-.489(.106)	-.493(.105)
other	-.121(.783)	-.054(.718)	-.043(.717)
NE	0	0	0
SE	-.033(.051)	-.056(.081)	-.050(.095)
Cent	-.245(.051)	-.204(.080)	-.203(.098)
West	-.209(.046)	-.204(.074)	-.184(.089)
extru	0	0	0
lomet	-.224(.083)	-.310(.127)	-.300(.126)
himet	.508(.081)	.511(.125)	.471(.127)
manct	.141(.079)	.143(.119)	.137(.123)
urbfr	.161(.081)	.188(.125)	.179(.125)
medct	.079(.074)	.113(.112)	.069(.113)
smp1c	-.066(.072)	-.038(.108)	-.052(.107)
nfnhs	0	0	0
finhs	-.105(.159)	-.033(.216)	.017(.213)
smcol	.120(.146)	.186(.208)	.234(.205)
colgr	.593(.154)	.607(.214)	.655(.211)
DK	.591(.142)	.588(.206)	.636(.202)
nores	.165(.141)	.221(.205)	.270(.202)
s^2_PSU	0	0	.036(.013)
s^2_sch	0	.154(.019)	.116(.019)
s^2	1.311(.039)	1.168(.037)	1.169(.037)
logL	-40,475.22	-40,349.04	-40,342.85

MIMIC estimates and SEs for the 2-, 3- and 4-level models

	2-level	3-level	4-level
male	0	0	0
femal	.121(.033)	.124(.032)	.124(.032)
white	0	0	0
black	-.780(.090)	-.652(.082)	-.657(.082)
hispa	-.526(.069)	-.464(.065)	-.462(.065)
as/pa	-.188(.117)	-.167(.126)	-.176(.127)
amind	-.619(.121)	-.487(.117)	-.494(.115)
other	.230(.672)	.167(.717)	.198(.721)
NE	0	0	0
SE	-.045(.049)	-.077(.078)	-.071(.088)
Cent	-.230(.055)	-.196(.077)	-.180(.096)
West	-.213(.049)	-.194(.071)	-.184(.083)
extru	0	0	0
lomet	-.207(.085)	-.319(.127)	-.332(.123)
himet	.505(.094)	.447(.133)	.408(.123)
manct	.174(.079)	.131(.120)	.134(.114)
urbfr	.173(.082)	.171(.125)	.167(.117)
medct	.079(.073)	.098(.114)	.069(.108)
smp1c	-.036(.072)	-.045(.109)	-.064(.100)
nfnhs	0	0	0
finhs	-.086(.154)	-.020(.219)	.018(.211)
smcol	.145(.140)	.168(.213)	.203(.205)
colgr	.595(.160)	.570(.225)	.608(.219)
DK	.594(.149)	.554(.218)	.592(.211)
nores	.194(.136)	.209(.211)	.245(.203)
s^2_PSU	0	0	.032(.014)
s^2_sch	0	.135(.032)	.108(.028)
s^2	1.185(.244)	1.065(.219)	1.078(.221)
logL	-40,080.32	-39,961.68	-39,954.46

2PL estimates and SEs for the 2-, 3- and 4-level models

	2-level	3-level	4-level
male	0	0	0
femal	.015(.028)	.012(.028)	.012(.028)
white	0	0	0
black	-.792(.041)	-.667(.047)	-.670(.046)
hispa	-.514(.041)	-.460(.043)	-.454(.043)
as/pa	-.240(.114)	-.203(.117)	-.206(.116)
amind	-.588(.092)	-.471(.093)	-.471(.093)
other	-.380(.855)	-.200(.752)	-.174(.739)
NE	0	0	0
SE	.003(.046)	-.020(.077)	-.036(.080)
Cent	-.219(.046)	-.172(.074)	-.176(.087)
West	-.193(.042)	-.182(.069)	-.198(.077)
extru	0	0	0
lomet	-.126(.075)	-.201(.113)	-.189(.120)
himet	.475(.074)	.497(.116)	.481(.119)
manct	.133(.072)	.150(.106)	.188(.121)
urbfr	.121(.074)	.158(.112)	.176(.122)
medct	.031(.067)	.092(.097)	.061(.110)
smplc	-.055(.066)	-.019(.095)	-.026(.102)
nfnhs	0	0	0
finhs	-.205(.152)	-.179(.206)	-.153(.201)
smcol	.061(.141)	.045(.200)	.065(.194)
colgr	.460(.149)	.398(.205)	.422(.199)
DK	.453(.138)	.382(.198)	.403(.192)
nores	.047(.137)	.027(.197)	.048(.191)
s ² _PSU	0	0	.019(.010)
s ² _sch	0	.139(.017)	.119(.017)
s ²	1.646(.345)	1.682(.365)	1.653(.360)
logL	-40,077.26	-39,930.05	-39,924.60

15 Appendix 2 – item parameter estimates

Rasch model intercepts

Item	null 2-level alpha (SE)	full 2-level alpha (SE)	full 3-level alpha (SE)	full 4-level alpha (SE)
1	3.391 (.090)	3.398 (.180)	3.400 (.248)	3.400 (.247)
2	1.735 (.058)	1.747 (.166)	1.749 (.238)	1.749 (.238)
3	-.384 (.049)	-.373 (.163)	-.372 (.236)	-.372 (.236)
4	.161 (.050)	.166 (.163)	.167 (.236)	.166 (.236)
5	-.029 (.050)	-.027 (.163)	-.027 (.236)	-.027 (.236)
6	-.578 (.052)	-.579 (.164)	-.579 (.236)	-.579 (.236)
7	-.014 (.052)	-.015 (.164)	-.016 (.236)	-.016 (.236)
8	-3.243 (.092)	-3.251 (.181)	-3.251 (.248)	-3.250 (.248)
9	.802 (.050)	.805 (.163)	.808 (.236)	.808 (.236)
10	.316 (.049)	.319 (.163)	.321 (.236)	.321 (.235)
11	.108 (.049)	.109 (.163)	.111 (.236)	.111 (.235)
12	.685 (.050)	.685 (.163)	.687 (.236)	.687 (.236)
13	-.668 (.050)	-.667 (.163)	-.666 (.236)	-.666 (.236)
14	-.425 (.050)	-.425 (.163)	-.425 (.236)	-.425 (.236)
15	-.215 (.050)	-.216 (.163)	-.216 (.236)	-.215 (.236)
16	.733 (.058)	.722 (.165)	.720 (.238)	.721 (.237)
17	-2.163 (.073)	-2.175 (.172)	-2.180 (.242)	-2.180 (.242)
18	-.987 (.062)	-1.000 (.167)	-1.005 (.238)	-1.004 (.238)
19	.915 (.064)	.885 (.168)	.876 (.239)	.876 (.239)
20	.479 (.065)	.448 (.169)	.434 (.239)	.434 (.239)
21	1.657 (.057)	1.669 (.165)	1.672 (.237)	1.673 (.237)
22	1.636 (.057)	1.647 (.165)	1.650 (.237)	1.651 (.237)
23	.338 (.049)	.349 (.163)	.353 (.236)	.353 (.235)
24	.976 (.052)	.987 (.164)	.990 (.236)	.990 (.236)
25	-.689 (.051)	-.684 (.164)	-.682 (.236)	-.682 (.236)
26	-.551 (.053)	-.549 (.164)	-.550 (.236)	-.550 (.236)
27	-2.097 (.067)	-2.100 (.169)	-2.101 (.240)	-2.101 (.240)
28	-1.358 (.064)	-1.360 (.169)	-1.358 (.239)	-1.359 (.239)
29	.408 (.049)	.421 (.163)	.422 (.235)	.421 (.235)
30	.024 (.051)	.027 (.163)	.024 (.236)	.024 (.236)
origin	0	-.020	-.118	-.077

MIMIC model slopes

Item	null 2-level beta (SE)	full 2-level beta (SE)	full 3-level beta (SE)	full 4-level beta (SE)
1	1	1	1	1
2	1.015 (.122)	1.037 (.125)	1.035 (.124)	1.025 (.123)
3	.898 (.109)	.908 (.110)	.899 (.109)	.889 (.107)
4	.992 (.121)	1.149 (.140)	1.144 (.138)	1.137 (.137)
5	.848 (.104)	.945 (.115)	.945 (.115)	.940 (.114)
6	1.023 (.126)	1.079 (.132)	1.101 (.134)	1.096 (.133)
7	.952 (.116)	.965 (.118)	.987 (.120)	.982 (.119)
8	1.140 (.169)	1.125 (.165)	1.149 (.169)	1.145 (.168)
9	.696 (.085)	.732 (.090)	.727 (.090)	.724 (.089)
10	.807 (.096)	.893 (.106)	.910 (.108)	.906 (.107)
11	.807 (.096)	.861 (.103)	.858 (.102)	.852 (.101)
12	1.897 (.219)	1.951 (.228)	1.967 (.231)	1.951 (.228)
13	2.170 (.264)	2.133 (.253)	2.112 (.251)	2.098 (.248)
14	1.885 (.221)	1.874 (.219)	1.872 (.219)	1.857 (.217)
15	.507 (.066)	.561 (.073)	.564 (.073)	.560 (.072)
16	1.005 (.121)	1.093 (.132)	1.131 (.136)	1.126 (.135)
17	1.135 (.146)	1.206 (.155)	1.229 (.158)	1.223 (.157)
18	.954 (.117)	1.045 (.129)	1.055 (.130)	1.050 (.129)
19	1.159 (.155)	1.325 (.175)	1.325 (.173)	1.316 (.171)
20	1.099 (.149)	1.169 (.157)	1.185 (.158)	1.178 (.157)
21	1.796 (.223)	1.665 (.201)	1.609 (.194)	1.597 (.192)
22	1.845 (.227)	1.735 (.209)	1.662 (.200)	1.651 (.198)
23	1.332 (.157)	1.426 (.169)	1.414 (.167)	1.402 (.165)
24	1.009 (.120)	1.132 (.135)	1.135 (.135)	1.129 (.134)
25	.977 (.118)	1.114 (.135)	1.171 (.141)	1.165 (.140)
26	1.002 (.123)	1.082 (.132)	1.083 (.132)	1.075 (.131)
27	.219 (.059)	.254 (.063)	.260 (.063)	.258 (.062)
28	.405 (.087)	.485 (.089)	.511 (.090)	.506 (.089)
29	.564 (.084)	.604 (.086)	.657 (.091)	.658 (.091)
30	.617 (.090)	.660 (.094)	.634 (.090)	.629 (.089)

2PL model slopes

Item	null 2-level beta (SE)	full 2-level beta (SE)	full 3-level beta (SE)	full 4-level beta (SE)
1 1		1	1	1
2	1.015 (.122)	1.012 (.126)	.972 (.126)	.978 (.127)
3	.898 (.109)	.926 (.116)	.915 (.121)	.929 (.123)
4	.992 (.121)	.715 (.093)	.668 (.091)	.675 (.092)
5	.848 (.104)	.653 (.086)	.620 (.086)	.628 (.087)
6	1.023 (.126)	.831 (.107)	.744 (.101)	.750 (.102)
7	.952 (.116)	.958 (.123)	.851 (.115)	.857 (.116)
8	1.140 (.169)	1.096 (.171)	1.027 (.169)	1.026 (.169)
9	.696 (.085)	.635 (.081)	.591 (.080)	.597 (.081)
10	.807 (.096)	.722 (.089)	.628 (.082)	.633 (.083)
11	.807 (.096)	.753 (.093)	.697 (.090)	.704 (.091)
12	1.897 (.219)	1.757 (.210)	1.735 (.216)	1.751 (.219)
13	2.170 (.264)	1.859 (.227)	1.943 (.251)	1.959 (.253)
14	1.885 (.221)	1.705 (.205)	1.787 (.224)	1.800 (.226)
15	.507 (.066)	.421 (.060)	.358 (.057)	.359 (.057)
16	1.005 (.121)	.944 (.118)	.767 (.103)	.772 (.104)
17	1.135 (.146)	1.011 (.137)	.853 (.124)	.857 (.125)
18	.954 (.117)	.764 (.100)	.690 (.096)	.696 (.097)
19	1.159 (.155)	.989 (.138)	.911 (.133)	.922 (.134)
20	1.099 (.149)	.946 (.134)	.876 (.131)	.883 (.132)
21	1.796 (.223)	1.847 (.236)	1.923 (.255)	1.932 (.256)
22	1.845 (.227)	1.824 (.231)	1.934 (.256)	1.951 (.259)
23	1.332 (.157)	1.161 (.141)	1.109 (.139)	1.119 (.141)
24	1.009 (.120)	.841 (.105)	.796 (.103)	.803 (.104)
25	.977 (.118)	.788 (.101)	.672 (.091)	.676 (.092)
26	1.002 (.123)	.857 (.110)	.774 (.104)	.780 (.105)
27	.219 (.059)	.128 (.057)	.091 (.057)	.092 (.058)
28	.405 (.087)	.320 (.085)	.265 (.084)	.267 (.085)
29	.564 (.084)	.537 (.084)	.406 (.073)	.409 (.074)
30	.617 (.090)	.563 (.087)	.552 (.091)	.555 (.092)

16 Appendix 3 – ETS and ML item estimates and SEs

item	ETS			ML	
	a	b	c	a	b
1	.503 (.019)	-3.780 (.143)	-	1.0	-3.255
2	.769 (.017)	-2.066 (.049)	-	1.035 (.124)	-1.588
3	.841 (.018)	-.642 (.019)	-	.899 (.109)	.488
4	1.090 (.045)	-.724 (.044)	.238	1.144 (.138)	-.054
5	.855 (.023)	-.860 (.032)	-	.945 (.115)	.121
6	1.150 (.065)	-.295 (.040)	.208	1.101 (.134)	.634
7	1.162 (.022)	-.848 (.024)	-	.987 (.120)	.111
8	1.738 (.028)	.125 (.018)	-	1.149 (.169)	3.027
9	.894 (.032)	-.960 (.044)	.280	.727 (.090)	-.875
10	.898 (.047)	-.715 (.050)	.352	.910 (.108)	-.223
11	.886 (.020)	-.900 (.028)	-	.858 (.102)	-.011
12	1.288 (.021)	-1.101 (.025)	-	1.967 (.231)	-.362
13	1.300 (.025)	-.445 (.017)	-	2.112 (.251)	.576
14	1.234 (.023)	-.554 (.018)	-	1.872 (.219)	.424
15	.620 (.037)	-.256 (.032)	.225	.564 (.073)	.416
16	.942 (.022)	-1.273 (.039)	-	1.131 (.136)	-.544
17	1.202 (.059)	.299 (.034)	-	1.229 (.158)	1.978
18	.865 (.023)	-.047 (.014)	-	1.055 (.130)	1.046
19	1.058 (.038)	-1.152 (.053)	.198	1.325 (.173)	-.615
20	1.101 (.053)	-.817 (.055)	.257	1.185 (.158)	-.275
21	.899 (.014)	-1.871 (.034)	-	1.609 (.194)	-1.181
22	.893 (.014)	-1.839 (.033)	-	1.662 (.200)	-1.148
23	1.017 (.016)	-1.042 (.021)	-	1.414 (.167)	-.204
24	1.185 (.027)	-1.074 (.034)	.232	1.135 (.135)	-.809
25	1.096 (.025)	-.376 (.020)	-	1.171 (.141)	.687
26	.998 (.024)	-.484 (.021)	-	1.083 (.132)	.591
27	1.766 (.296)	1.115 (.248)	.197	.260 (.063)	6.512
28	1.149 (.034)	.365 (.021)	.164	.511 (.090)	2.333
29	.955 (.044)	-.544 (.040)	.247	.657 (.091)	-.449
30	.974 (.051)	-.454 (.042)	.243	.634 (.090)	.072

17 Appendix 4 – parameter correlation matrix for 3-level Rasch model

	i1	i2	i3	i4	i5
i1	1.0				
i2	.913	1.0			
i3	.917	.956	1.0		
i4	.918	.956	.964	1.0	
i5	.917	.955	.964	.963	1.0
i6	.915	.953	.962	.962	.961
i7	.916	.954	.962	.962	.962
i8	.868	.906	.916	.915	.915
i9	.913	.951	.958	.958	.958
i10	.914	.952	.960	.959	.959
i11	.914	.952	.960	.959	.959
i12	.913	.951	.958	.958	.958
i13	.912	.950	.958	.958	.958
i14	.913	.951	.959	.958	.958
i15	.913	.951	.959	.959	.958
i16	.907	.944	.952	.951	.951
i17	.889	.927	.935	.935	.934
i18	.903	.940	.949	.948	.948
i19	.901	.938	.946	.946	.945
i20	.899	.936	.944	.944	.943
i21	.908	.945	.952	.952	.951
i22	.908	.945	.952	.952	.952
i23	.914	.952	.959	.959	.958
i24	.912	.950	.957	.957	.956
i25	.911	.949	.957	.957	.956
i26	.910	.948	.956	.956	.955
i27	.895	.933	.942	.941	.941
i28	.899	.937	.945	.945	.944
i29	.914	.952	.960	.959	.959
i30	.912	.950	.958	.957	.957
sex2	-.056	-.060	-.061	-.061	-.061
race2	-.037	-.035	-.030	-.031	-.030
race3	-.022	-.021	-.018	-.018	-.018
race4	-.033	-.034	-.033	-.033	-.033
race5	-.027	-.026	-.025	-.025	-.025
race6	-.053	-.054	-.053	-.053	-.053

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regi2 -.223  -.232  -.234  -.234  -.234
regi3 -.243  -.252  -.253  -.253  -.253
regi4 -.216  -.224  -.225  -.226  -.225

stoc2 -.325  -.338  -.339  -.339  -.339
stoc3 -.318  -.332  -.336  -.335  -.335
stoc4 -.280  -.292  -.294  -.294  -.294
stoc5 -.337  -.351  -.354  -.354  -.354
stoc6 -.300  -.312  -.315  -.315  -.315
stoc7 -.341  -.355  -.358  -.358  -.357

pare2 -.765  -.796  -.803  -.803  -.802
pare3 -.794  -.827  -.834  -.834  -.834
pare4 -.771  -.804  -.811  -.811  -.811
pare5 -.801  -.835  -.843  -.843  -.842
pare6 -.803  -.837  -.844  -.844  -.844

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-----
                i6      i7      i8      i9      i10
-----
i6      1.0
i7      .960  1.0
i8      .914  .914  1.0
i9      .956  .956  .910  1.0
i10     .957  .957  .911  .964  1.0
i11     .957  .957  .911  .963  .965
i12     .956  .956  .910  .962  .963
i13     .956  .956  .911  .962  .963
i14     .957  .957  .911  .963  .964
i15     .957  .957  .911  .963  .964
i16     .949  .950  .903  .956  .957
i17     .933  .933  .890  .938  .940
i18     .946  .946  .901  .952  .953
i19     .944  .944  .898  .945  .946
i20     .942  .942  .896  .943  .944
i21     .950  .950  .903  .952  .953
i22     .950  .950  .903  .952  .953
i23     .957  .957  .911  .958  .959
i24     .955  .955  .908  .956  .957
i25     .955  .955  .910  .956  .957
i26     .954  .954  .908  .955  .956
i27     .940  .940  .896  .940  .942
i28     .943  .943  .899  .944  .945
i29     .957  .958  .911  .959  .960

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i30	.955	.956	.909	.957	.958
sex2	-.061	-.061	-.059	-.058	-.058
race2	-.029	-.030	-.023	-.032	-.031
race3	-.017	-.018	-.012	-.019	-.019
race4	-.033	-.033	-.031	-.034	-.034
race5	-.025	-.025	-.021	-.024	-.024
race6	-.053	-.053	-.050	-.054	-.054
regi2	-.233	-.233	-.222	-.234	-.235
regi3	-.252	-.253	-.239	-.254	-.254
regi4	-.224	-.225	-.213	-.226	-.226
stoc2	-.338	-.338	-.321	-.340	-.340
stoc3	-.335	-.335	-.321	-.335	-.336
stoc4	-.293	-.293	-.280	-.293	-.294
stoc5	-.353	-.353	-.337	-.354	-.354
stoc6	-.315	-.314	-.300	-.315	-.316
stoc7	-.357	-.357	-.339	-.358	-.358
pare2	-.801	-.801	-.762	-.801	-.802
pare3	-.832	-.833	-.792	-.833	-.834
pare4	-.810	-.810	-.772	-.810	-.812
pare5	-.841	-.841	-.802	-.842	-.843
pare6	-.843	-.843	-.802	-.843	-.845

	i11	i12	i13	i14	i15
i11	1.0				
i12	.964	1.0			
i13	.964	.962	1.0		
i14	.964	.963	.963	1.0	
i15	.964	.963	.963	.964	1.0
i16	.957	.956	.955	.956	.956
i17	.940	.938	.939	.940	.940
i18	.953	.952	.953	.953	.953
i19	.946	.945	.945	.945	.945
i20	.944	.943	.944	.944	.944
i21	.953	.951	.951	.952	.952
i22	.953	.952	.951	.952	.952
i23	.959	.958	.958	.959	.959
i24	.957	.956	.956	.957	.957
i25	.956	.956	.956	.957	.957

i26	.956	.955	.956	.956	.956
i27	.942	.940	.941	.941	.941
i28	.945	.944	.944	.945	.945
i29	.960	.959	.959	.959	.959
i30	.958	.957	.957	.957	.957
sex2	-.059	-.058	-.059	-.059	-.059
race2	-.030	-.031	-.029	-.029	-.029
race3	-.018	-.019	-.017	-.018	-.018
race4	-.034	-.034	-.034	-.034	-.034
race5	-.024	-.024	-.024	-.024	-.024
race6	-.054	-.054	-.054	-.054	-.054
regi2	-.234	-.234	-.234	-.234	-.234
regi3	-.254	-.254	-.253	-.253	-.254
regi4	-.226	-.226	-.225	-.225	-.225
stoc2	-.340	-.340	-.340	-.340	-.340
stoc3	-.336	-.336	-.336	-.336	-.336
stoc4	-.294	-.293	-.294	-.294	-.294
stoc5	-.354	-.354	-.354	-.354	-.355
stoc6	-.316	-.315	-.315	-.315	-.316
stoc7	-.358	-.358	-.358	-.358	-.358
pare2	-.803	-.802	-.802	-.802	-.802
pare3	-.834	-.833	-.833	-.835	-.834
pare4	-.812	-.810	-.811	-.811	-.811
pare5	-.843	-.842	-.842	-.843	-.843
pare6	-.845	-.844	-.844	-.844	-.844

	i16	i17	i18	i19	i20

i16	1.0				
i17	.932	1.0			
i18	.946	.931	1.0		
i19	.938	.922	.935	1.0	
i20	.937	.920	.933	.938	1.0
i21	.945	.927	.941	.939	.937
i22	.945	.927	.941	.939	.937
i23	.952	.935	.948	.946	.944
i24	.950	.932	.946	.944	.942
i25	.950	.933	.947	.944	.942
i26	.948	.932	.945	.942	.941

i27	.934	.919	.932	.928	.927
i28	.937	.922	.935	.931	.930
i29	.952	.935	.949	.946	.944
i30	.950	.933	.947	.944	.942
sex2	-.058	-.058	-.059	-.060	-.061
race2	-.030	-.025	-.028	-.031	-.029
race3	-.019	-.015	-.017	-.021	-.020
race4	-.034	-.034	-.035	-.033	-.033
race5	-.024	-.022	-.023	-.025	-.024
race6	-.054	-.053	-.054	-.054	-.054
regi2	-.233	-.228	-.231	-.232	-.231
regi3	-.252	-.246	-.250	-.251	-.250
regi4	-.224	-.218	-.222	-.222	-.222
stoc2	-.337	-.330	-.335	-.334	-.333
stoc3	-.333	-.329	-.333	-.332	-.332
stoc4	-.292	-.287	-.291	-.290	-.289
stoc5	-.352	-.346	-.351	-.350	-.350
stoc6	-.313	-.308	-.312	-.311	-.310
stoc7	-.355	-.349	-.354	-.353	-.352
pare2	-.796	-.782	-.793	-.791	-.789
pare3	-.827	-.813	-.825	-.822	-.820
pare4	-.805	-.792	-.803	-.799	-.798
pare5	-.836	-.822	-.834	-.830	-.829
pare6	-.838	-.823	-.835	-.832	-.830

	i21	i22	i23	i24	i25

i21	1.0				
i22	.952	1.0			
i23	.957	.957	1.0		
i24	.956	.956	.962	1.0	
i25	.954	.954	.962	.960	1.0
i26	.953	.953	.961	.958	.959
i27	.938	.938	.946	.943	.945
i28	.938	.938	.945	.942	.944
i29	.953	.953	.960	.958	.958
i30	.951	.951	.958	.955	.956
sex2	-.058	-.058	-.059	-.059	-.060

race2	-.035	-.035	-.032	-.033	-.029
race3	-.024	-.023	-.021	-.022	-.019
race4	-.036	-.036	-.035	-.035	-.034
race5	-.024	-.024	-.024	-.024	-.023
race6	-.052	-.052	-.053	-.052	-.052
regi2	-.233	-.233	-.235	-.234	-.234
regi3	-.253	-.253	-.254	-.253	-.253
regi4	-.224	-.224	-.225	-.225	-.224
stoc2	-.340	-.340	-.341	-.341	-.340
stoc3	-.334	-.337	-.337	-.336	-.336
stoc4	-.293	-.293	-.295	-.295	-.295
stoc5	-.352	-.352	-.355	-.354	-.354
stoc6	-.315	-.315	-.317	-.316	-.316
stoc7	-.357	-.357	-.359	-.359	-.358
pare2	-.796	-.796	-.802	-.800	-.800
pare3	-.827	-.827	-.833	-.831	-.832
pare4	-.803	-.804	-.810	-.808	-.810
pare5	-.835	-.835	-.842	-.840	-.841
pare6	-.837	-.837	-.843	-.841	-.842

	i26	i27	i28	i29	i30

i26	1.0				
i27	.944	1.0			
i28	.942	.929	1.0		
i29	.956	.942	.945	1.0	
i30	.954	.940	.943	.962	1.0
sex2	-.060	-.060	-.058	-.058	-.058
race2	-.029	-.025	-.027	-.032	-.030
race3	-.019	-.016	-.016	-.020	-.019
race4	-.034	-.033	-.033	-.033	-.033
race5	-.023	-.022	-.025	-.026	-.026
race6	-.052	-.051	-.055	-.053	-.052
regi2	-.234	-.230	-.231	-.235	-.234
regi3	-.252	-.248	-.250	-.254	-.253
regi4	-.224	-.220	-.222	-.226	-.225

stoc2	-.339	-.334	-.334	-.341	-.339
stoc3	-.336	-.332	-.333	-.336	-.336
stoc4	-.294	-.290	-.290	-.295	-.294
stoc5	-.354	-.349	-.350	-.354	-.354
stoc6	-.316	-.311	-.312	-.316	-.315
stoc7	-.358	-.352	-.353	-.358	-.358

pare2	-.799	-.787	-.790	-.802	-.801
pare3	-.831	-.818	-.821	-.834	-.833
pare4	-.808	-.797	-.800	-.811	-.809
pare5	-.840	-.828	-.831	-.843	-.841
pare6	-.841	-.828	-.831	-.844	-.842

 sex2 race2 race3 race4 race5

sex2	1.0				
race2	-.015	1.0			
race3	.041	.266	1.0		
race4	.015	.106	.115	1.0	
race5	.022	.122	.121	.043	1.0
race6	.012	.012	.018	.010	.007

regi2	.001	-.127	-.020	-.027	-.003
regi3	.009	-.008	.010	-.011	.021
regi4	.005	.021	-.103	-.047	-.026

stoc2	-.003	-.197	-.107	-.055	-.031
stoc3	.016	-.050	-.001	-.008	.003
stoc4	.011	-.120	-.062	-.034	-.008
stoc5	.015	-.037	-.015	-.006	.014
stoc6	.009	-.076	-.030	-.003	.003
stoc7	.004	-.041	-.019	-.003	.007

pare2	-.013	.034	-.005	.035	.014
pare3	-.008	.031	-.005	.038	.006
pare4	-.006	.030	-.003	.031	.007
pare5	-.006	.028	.001	.035	.004
pare6	-.017	.036	-.002	.037	.006

 race6 regi2 regi3 regi4 stoc2

race6	1.0				
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regi2	.002	1.0			
regi3	-.004	.554	1.0		
regi4	-.003	.568	.575	1.0	
stoc2	-.005	.205	.231	.105	1.0
stoc3	-.001	.106	.192	.040	.603
stoc4	-.008	.054	.126	-.058	.637
stoc5	-.001	.184	.242	.097	.619
stoc6	-.008	-.017	.147	.017	.650
stoc7	-.006	.053	.218	.050	.688
pare2	.060	.011	-.019	.019	-.017
pare3	.059	.019	-.026	.018	-.011
pare4	.060	.018	-.020	.020	-.012
pare5	.063	.025	-.019	.025	-.013
pare6	.062	.020	-.022	.018	-.017

	stoc3	stoc4	stoc5	stoc6	stoc7

stoc3	1.0				
stoc4	.628	1.0			
stoc5	.612	.631	1.0		
stoc6	.656	.690	.655	1.0	
stoc7	.690	.720	.697	.764	1.0
pare2	-.000	-.040	-.006	-.039	-.032
pare3	.000	-.040	-.005	-.043	-.034
pare4	-.001	-.037	-.007	-.041	-.028
pare5	-.015	-.047	-.012	-.050	-.037
pare6	-.009	-.049	-.012	-.048	-.038

	pare2	pare3	pare4	pare5	pare6

pared2	1.0				
pared3	.923	1.0			
pared4	.897	.931	1.0		
pared5	.933	.969	.942	1.0	
pared6	.937	.973	.945	.984	1.0

18 Appendix 5 – parameter correlation matrix for 4-level 2PL model

Item intercepts

	i1	i2	i3	i4	i5
i 1	1.0				
i 2	.778	1.0			
i 3	.799	.926	1.0		
i 4	.799	.928	.959	1.0	
i 5	.799	.927	.958	.961	1.0
i 6	.797	.925	.955	.957	.957
i 7	.797	.925	.956	.958	.957
i 8	.652	.757	.776	.779	.779
i 9	.795	.922	.953	.956	.957
i10	.796	.924	.954	.958	.958
i11	.796	.923	.954	.958	.958
i12	.772	.896	.925	.928	.928
i13	.751	.871	.899	.902	.902
i14	.772	.894	.924	.926	.926
i15	.797	.925	.956	.960	.960
i16	.786	.913	.943	.946	.947
i17	.747	.867	.895	.899	.899
i18	.783	.909	.940	.943	.943
i19	.775	.900	.929	.933	.933
i20	.777	.903	.933	.936	.936
i21	.689	.797	.825	.827	.827
i22	.690	.801	.831	.833	.833
i23	.789	.916	.946	.950	.950
i24	.789	.916	.946	.905	.950
i25	.793	.920	.951	.955	.955
i26	.791	.918	.948	.952	.952
i27	.783	.909	.940	.943	.943
i28	.785	.911	.942	.946	.946
i29	.797	.925	.956	.959	.960
i30	.795	.922	.953	.957	.957
sex2	-.057	-.068	-.060	-.062	-.063
race2	-.031	-.035	-.029	-.030	-.030
race3	-.021	-.022	-.018	-.018	-.018

race4	-.032	-.032	-.034	-.035	-.035
race5	-.023	-.020	-.023	-.024	-.024
race6	-.046	-.047	-.051	-.051	-.051
regi2	-.195	-.217	-.222	-.223	-.223
regi3	-.214	-.249	-.250	-.251	-.251
regi4	-.186	-.210	-.212	-.213	-.213
stoc2	-.297	-.342	-.353	-.355	-.355
stoc3	-.290	-.341	-.355	-.356	-.356
stoc4	-.261	-.303	-.314	-.316	-.315
stoc5	-.275	-.324	-.335	-.336	-.336
stoc6	-.265	-.310	-.320	-.322	-.322
stoc7	-.304	-.355	-.366	-.368	-.368
pare2	-.654	-.760	-.788	-.791	-.791
pare3	-.679	-.788	-.818	-.821	-.821
pare4	-.657	-.765	-.796	-.799	-.799
pare5	-.684	-.794	-.824	-.827	-.827
pare6	-.687	-.797	-.826	-.829	-.829

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-----
                i6      i7      i8      i9      i10
-----
i6      1.0
i7      .954  1.0
i8      .776  .778  1.0
i9      .952  .952  .774  1.0
i10     .954  .954  .775  .960  1.0
i11     .953  .953  .775  .960  .962
i12     .924  .924  .751  .934  .936
i13     .898  .898  .728  .911  .913
i14     .923  .923  .750  .934  .936
i15     .956  .955  .777  .961  .967
i16     .942  .942  .766  .949  .951
i17     .895  .895  .728  .901  .903
i18     .939  .939  .762  .945  .947
i19     .929  .929  .758  .932  .933
i20     .932  .932  .759  .935  .937
i21     .824  .824  .670  .827  .828
i22     .830  .830  .674  .833  .835
i23     .946  .946  .769  .949  .951
i24     .946  .946  .769  .949  .951
i25     .950  .950  .773  .954  .956
i26     .948  .948  .770  .951  .953

```

i27	.939	.939	.763	.942	.944
i28	.942	.941	.766	.945	.947
i29	.955	.955	.776	.959	.960
i30	.953	.953	.774	.956	.958
sex2	-.065	-.063	-.045	-.059	-.060
race2	-.029	-.030	-.013	-.033	-.031
race3	-.017	-.018	-.012	-.021	-.020
race4	-.035	-.035	-.024	-.037	-.037
race5	-.023	-.023	-.016	-.026	-.024
race6	-.051	-.051	-.040	-.050	-.051
regi2	-.222	-.222	-.181	-.224	-.224
regi3	-.250	-.250	-.212	-.253	-.252
regi4	-.212	-.212	-.177	-.214	-.214
stoc2	-.353	-.353	-.289	-.356	-.356
stoc3	-.354	-.354	-.291	-.354	-.356
stoc4	-.314	-.314	-.258	-.315	-.315
stoc5	-.334	-.334	-.272	-.336	-.337
stoc6	-.320	-.320	-.263	-.321	-.322
stoc7	-.365	-.365	-.300	-.368	-.368
pare2	-.788	-.788	-.639	-.790	-.791
pare3	-.818	-.817	-.662	-.820	-.821
pare4	-.795	-.795	-.645	-.798	-.799
pare5	-.824	-.823	-.667	-.826	-.827
pare6	-.825	-.825	-.669	-.828	-.830

	i11	i12	i13	i14	i15

i11	1.0				
i12	.937	1.0			
i13	.913	.899	1.0		
i14	.937	.923	.911	1.0	
i15	.962	.934	.909	.933	1.0
i16	.951	.927	.906	.928	.951
i17	.903	.883	.856	.879	.903
i18	.947	.923	.898	.921	.948
i19	.933	.905	.880	.904	.935
i20	.937	.908	.883	.907	.939
i21	.828	.807	.787	.808	.829
i22	.834	.812	.792	.813	.835

i23	.950	.923	.897	.921	.952
i24	.950	.921	.896	.920	.952
i25	.955	.926	.900	.924	.957
i26	.953	.924	.898	.922	.955
i27	.944	.913	.887	.911	.946
i28	.946	.916	.889	.914	.949
i29	.960	.929	.904	.928	.962
i30	.957	.927	.902	.926	.959
sex2	-.060	-.067	-.047	-.052	-.060
race2	-.030	-.024	-.042	-.035	-.028
race3	-.019	-.013	-.025	-.026	-.018
race4	-.037	-.036	-.041	-.038	-.037
race5	-.023	-.018	-.030	-.024	-.023
race6	-.051	-.050	-.049	-.050	-.052
regi2	-.224	-.214	-.214	-.219	-.224
regi3	-.252	-.246	-.243	-.247	-.252
regi4	-.214	-.206	-.207	-.211	-.214
stoc2	-.356	-.341	-.345	-.346	-.356
stoc3	-.355	-.347	-.333	-.343	-.356
stoc4	-.315	-.306	-.301	-.304	-.315
stoc5	-.337	-.328	-.320	-.325	-.337
stoc6	-.322	-.315	-.309	-.313	-.323
stoc7	-.367	-.358	-.352	-.358	-.369
pare2	-.791	-.765	-.739	-.762	-.793
pare3	-.821	-.792	-.767	-.790	-.824
pare4	-.799	-.773	-.744	-.768	-.802
pare5	-.827	-.798	-.770	-.795	-.830
pare6	-.829	-.801	-.774	-.798	-.832

	i16	i17	i18	i19	i20
i16	1.0				
i17	.892	1.0			
i18	.937	.891	1.0		
i19	.922	.876	.919	1.0	
i20	.926	.879	.922	.920	1.0
i21	.819	.777	.815	.807	.811
i22	.824	.783	.821	.812	.817
i23	.940	.892	.936	.926	.929

i24	.939	.892	.936	.926	.929
i25	.944	.897	.941	.930	.934
i26	.942	.894	.938	.928	.931
i27	.933	.885	.929	.918	.922
i28	.935	.888	.932	.921	.925
i29	.949	.900	.945	.934	.938
i30	.946	.898	.942	.931	.935
sex2	-.059	-.065	-.064	-.062	-.061
race2	-.032	-.019	-.028	-.033	-.029
race3	-.020	-.013	-.017	-.018	-.020
race4	-.037	-.030	-.036	-.036	-.034
race5	-.024	-.017	-.022	-.020	-.022
race6	-.052	-.044	-.053	-.052	-.052
regi2	-.220	-.209	-.220	-.216	-.218
regi3	-.250	-.237	-.247	-.245	-.245
regi4	-.211	-.198	-.209	-.208	-.208
stoc2	-.351	-.333	-.349	-.343	-.346
stoc3	-.351	-.332	-.351	-.345	-.348
stoc4	-.311	-.295	-.311	-.304	-.307
stoc5	-.333	-.316	-.332	-.326	-.330
stoc6	-.319	-.298	-.317	-.313	-.315
stoc7	-.364	-.344	-.362	-.357	-.359
pare2	-.781	-.743	-.779	-.770	-.773
pare3	-.811	-.772	-.808	-.800	-.802
pare4	-.788	-.751	-.787	-.778	-.780
pare5	-.816	-.778	-.815	-.806	-.808
pare6	-.819	-.779	-.816	-.808	-.810

	i21	i22	i23	i24	i25
i21	1.0				
i22	.780	1.0			
i23	.836	.844	1.0		
i24	.829	.836	.950	1.0	
i25	.835	.841	.953	.951	1.0
i26	.836	.841	.952	.949	.954
i27	.815	.822	.936	.936	.942
i28	.817	.824	.938	.938	.943
i29	.829	.835	.952	.952	.957

i30	.827	.833	.949	.949	.954
sex2	-.056	-.060	-.062	-.060	-.061
race2	-.026	-.017	-.029	-.032	-.028
race3	-.024	-.018	-.020	-.021	-.019
race4	-.041	-.030	-.035	-.035	-.036
race5	-.025	-.023	-.023	-.024	-.022
race6	-.044	-.045	-.050	-.049	-.050
regi2	-.202	-.207	-.221	-.220	-.223
regi3	-.216	-.212	-.249	-.250	-.250
regi4	-.190	-.192	-.212	-.213	-.213
stoc2	-.299	-.301	-.352	-.354	-.355
stoc3	-.299	-.304	-.354	-.353	-.357
stoc4	-.265	-.270	-.315	-.315	-.317
stoc5	-.278	-.277	-.334	-.335	-.336
stoc6	-.270	-.274	-.320	-.321	-.322
stoc7	-.309	-.310	-.365	-.366	-.368
pare2	-.684	-.688	-.784	-.784	-.788
pare3	-.708	-.717	-.814	-.814	-.818
pare4	-.691	-.698	-.792	-.791	-.796
pare5	-.718	-.725	-.820	-.820	-.824
pare6	-.718	-.722	-.822	-.822	-.826

	i26	i27	i28	i29	i30

i26	1.0				
i27	.939	1.0			
i28	.941	.933	1.0		
i29	.954	.946	.948	1.0	
i30	.952	.943	.945	.961	1.0
sex2	-.063	-.062	-.058	-.060	-.060
race2	-.027	-.022	-.025	-.034	-.031
race3	-.018	-.014	-.014	-.021	-.020
race4	-.035	-.034	-.034	-.036	-.036
race5	-.022	-.020	-.023	-.026	-.025
race6	-.050	-.049	-.053	-.050	-.050
regi2	-.222	-.220	-.221	-.224	-.223

regi3	-.250	-.247	-.249	-.252	-.251
regi4	-.212	-.208	-.211	-.214	-.213
stoc2	-.355	-.351	-.350	-.357	-.356
stoc3	-.355	-.355	-.354	-.356	-.356
stoc4	-.316	-.314	-.312	-.317	-.316
stoc5	-.335	-.334	-.333	-.337	-.337
stoc6	-.321	-.319	-.318	-.322	-.322
stoc7	-.367	-.364	-.364	-.369	-.368
pare2	-.785	-.779	-.782	-.793	-.790
pare3	-.816	-.810	-.812	-.823	-.821
pare4	-.794	-.788	-.791	-.800	-.798
pare5	-.822	-.816	-.819	-.829	-.826
pare6	-.824	-.817	-.819	-.831	-.828

	sex2	race2	race3	race4	race5

sex2	1.0				
race2	-.010	1.0			
race3	.029	.259	1.0		
race4	.017	.100	.113	1.0	
race5	.020	.119	.126	.042	1.0
race6	.012	.013	.012	.009	.005
regi2	-.003	-.080	-.013	-.009	-.001
regi3	-.002	.001	.014	-.000	.009
regi4	.009	.023	-.069	-.026	-.032
stoc2	.000	-.139	-.108	-.034	-.017
stoc3	.003	-.088	-.021	-.008	.014
stoc4	.013	-.080	-.053	-.027	.008
stoc5	.011	-.040	-.008	-.006	.023
stoc6	.005	-.072	-.038	-.002	.015
stoc7	-.002	-.033	-.017	-.003	.022
pare2	-.005	.034	.004	.032	.006
pare3	-.000	.027	-.001	.034	-.003
pare4	.003	.025	.003	.027	-.001
pare5	.001	.026	.005	.032	-.004
pare6	-.008	.032	.005	.034	-.001

	race6	regi2	regi3	regi4	stoc2

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race6 1.0

regi2 -.001 1.0
regi3 -.008 .497 1.0
regi4 -.007 .543 .500 1.0

stoc2 -.005 .132 .219 .060 1.0
stoc3 -.009 .098 .243 .013 .680
stoc4 -.007 .064 .175 -.007 .677
stoc5 -.004 .100 .241 .009 .646
stoc6 -.007 -.049 .200 .013 .658
stoc7 -.006 .018 .245 .044 .713

pare2 .059 .003 -.039 .008 -.017
pare3 .058 .013 -.047 .008 -.005
pare4 .060 .009 -.042 .006 -.012
pare5 .062 .016 -.043 .011 -.010
pare6 .061 .010 -.045 .003 -.015
-----
          stoc3  stoc4  stoc5  stoc6  stoc7
-----
stoc3 1.0
stoc4 .665 1.0
stoc5 .687 .624 1.0
stoc6 .701 .650 .678 1.0
stoc7 .730 .701 .707 .783 1.0

pare2 -.022 -.030 -.034 -.059 -.045
pare3 -.020 -.029 -.034 -.061 -.044
pare4 -.021 -.027 -.037 -.058 -.041
pare5 -.038 -.038 -.045 -.070 -.050
pare6 -.031 -.040 -.043 -.066 -.049
-----
          pare2  pare3  pare4  pare5  pare6
-----
pare2 1.0
pare3 .928 1.0
pare4 .904 .935 1.0
pare5 .937 .972 .945 1.0
pare6 .941 .975 .948 .985 1.0
-----

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Item slopes

	i1	i2	i3	i4	i5
s2	-.389	.113	-.010	-.009	-.009
s3	-.393	-.005	-.027	-.003	-.003
s4	-.384	-.014	.001	.007	-.001
s5	-.383	-.022	-.010	-.010	-.012
s6	-.390	-.021	-.006	-.006	-.007
s7	-.392	-.016	-.004	-.003	-.003
s8	-.329	-.027	-.014	-.015	-.015
s9	-.383	-.012	-.007	-.005	-.006
s10	-.391	-.008	-.003	-.001	-.002
s11	-.397	-.009	-.004	-.002	-.003
s12	-.411	-.007	.000	.001	.001
s13	-.407	-.019	-.012	-.010	-.011
s14	-.419	-.016	-.009	-.007	-.007
s15	-.322	-.006	-.001	.001	.001
s16	-.378	.000	.006	.008	.007
s17	-.355	-.011	-.003	-.002	-.002
s18	-.372	-.014	-.008	-.006	-.007
s19	-.350	-.003	.000	.002	.001
s20	-.349	-.012	-.006	-.005	-.005
s21	-.377	.001	.010	.011	.010
s22	-.373	.010	.021	.022	.022
s23	-.412	-.013	-.007	-.005	-.005
s24	-.398	-.012	-.006	-.004	-.004
s25	-.389	-.019	-.012	-.011	-.011
s26	-.393	-.017	-.011	-.010	-.010
s27	-.070	.010	.012	.013	.013
s28	-.166	-.012	-.010	-.009	-.010
s29	-.301	-.015	-.009	-.009	-.009
s30	-.322	-.015	-.010	-.007	-.008
	i6	i7	i8	i9	i10
s2	-.008	-.009	-.001	-.008	-.008
s3	-.002	-.004	.013	-.002	-.003
s4	-.005	-.002	.006	-.001	-.000
s5	-.013	-.012	.003	-.011	-.011
s6	-.038	-.007	-.007	-.006	-.006
s7	-.003	-.004	-.039	-.003	-.003
s8	-.015	-.016	-.393	-.015	-.015

s9	-.007	-.006	-.012	.020	-.005
s10	-.003	-.002	-.010	-.000	.007
s11	-.004	-.003	-.011	-.002	-.002
s12	-.000	.001	-.006	-.002	-.001
s13	-.012	-.012	-.012	-.015	-.014
s14	-.009	-.008	-.013	-.010	-.009
s15	-.001	.000	-.008	.002	.001
s16	.006	.007	-.000	.008	.008
s17	-.003	-.003	-.011	-.001	-.001
s18	-.008	-.007	-.007	-.005	-.006
s19	.000	.001	.008	.001	.001
s20	-.007	-.005	-.007	-.004	-.004
s21	.009	.010	.001	.011	.011
s22	.020	.021	.010	.022	.022
s23	-.007	-.006	-.013	-.005	-.005
s24	-.006	-.005	-.010	-.004	-.004
s25	-.013	-.012	-.019	-.011	-.011
s26	-.012	-.011	-.017	-.010	-.010
s27	.012	.012	.005	.013	.013
s28	-.010	-.010	-.017	-.009	-.009
s29	-.011	-.010	-.017	-.009	-.009
s30	-.008	-.009	-.017	-.005	-.006

	i11	i12	i13	i14	i15
s2	-.008	-.007	-.007	-.009	-.008
s3	-.003	-.004	-.002	-.005	-.002
s4	-.000	-.001	-.004	-.002	.000
s5	-.011	-.011	-.013	-.013	-.010
s6	-.006	-.005	-.007	-.008	-.006
s7	-.003	-.004	-.002	-.004	-.003
s8	-.015	-.014	-.012	-.015	-.014
s9	-.006	-.014	.004	-.002	-.005
s10	-.001	-.004	.009	.000	-.001
s11	-.002	-.009	.008	.000	-.002
s12	-.006	.066	-.022	-.011	.001
s13	-.014	-.009	-.149	-.031	-.011
s14	-.009	-.008	-.026	-.080	-.007
s15	.001	-.005	.010	.004	-.008
s16	.008	.003	.020	.013	.008
s17	-.001	-.010	.005	-.001	-.001
s18	-.006	-.012	.001	-.004	-.005
s19	.001	.001	.002	.001	.001
s20	-.004	-.004	-.003	-.003	-.004

s21	.011	.010	.014	.012	.011
s22	.022	.020	.024	.022	.022
s23	-.005	-.007	-.009	-.010	-.004
s24	-.004	-.005	-.005	-.006	-.003
s25	-.011	-.014	-.010	-.014	-.010
s26	-.010	-.011	-.010	-.013	-.009
s27	.013	.012	.014	.013	.013
s28	-.009	-.009	-.003	-.006	-.010
s29	-.009	-.010	-.002	-.009	-.009
s30	-.006	-.009	-.001	-.003	-.006

	i16	i17	i18	i19	i20
s2	-.007	-.006	-.008	-.006	-.007
s3	-.002	-.002	-.002	-.000	-.002
s4	-.000	-.001	.000	.000	.000
s5	-.011	-.009	-.011	-.010	-.010
s6	-.006	-.006	-.005	-.007	-.005
s7	-.002	-.004	-.002	-.002	-.001
s8	-.014	-.015	-.013	-.018	-.015
s9	-.005	-.008	-.006	-.004	-.004
s10	.000	-.004	-.001	.000	.000
s11	-.002	-.005	-.002	-.001	-.001
s12	-.003	.007	.003	.001	.002
s13	-.018	-.006	-.011	-.010	-.010
s14	-.012	-.002	-.006	-.007	-.007
s15	.002	-.002	.001	.003	.002
s16	.044	-.007	.006	.009	.008
s17	.003	-.168	-.002	-.001	-.000
s18	-.003	-.011	-.056	-.005	-.005
s19	.002	.003	.002	.079	-.002
s20	-.004	-.003	-.004	-.000	.031
s21	.011	.009	.011	.012	.013
s22	.022	.019	.022	.022	.024
s23	-.005	-.006	-.004	-.004	-.003
s24	-.004	-.005	-.003	-.002	-.003
s25	-.010	-.012	-.010	-.010	-.010
s26	-.009	-.011	-.009	-.007	-.008
s27	.013	.012	.013	.012	.013
s28	-.009	-.009	-.010	-.010	-.010
s29	-.009	-.012	-.009	-.010	-.009
s30	-.005	-.008	-.006	-.007	-.006

	i21	i22	i23	i24	i25
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s2	-.014	-.009	-.008	-.008	-.008
s3	-.013	-.004	-.003	-.002	-.003
s4	-.002	.005	.000	.000	.000
s5	-.015	-.008	-.011	-.010	-.011
s6	-.011	-.003	-.006	-.006	-.006
s7	-.005	.001	-.003	-.003	-.003
s8	-.016	-.010	-.014	-.015	-.015
s9	-.009	-.002	-.005	-.004	-.005
s10	-.007	-.001	-.002	-.001	-.001
s11	-.007	-.001	-.002	-.002	-.003
s12	-.003	.004	.002	.001	.002
s13	-.019	-.012	-.011	-.011	-.010
s14	-.019	-.008	-.008	-.008	-.007
s15	-.002	.001	.001	.001	.001
s16	.002	.010	.007	.007	.008
s17	-.004	.002	-.002	-.002	-.002
s18	-.008	-.001	-.006	-.006	-.006
s19	-.010	-.006	.001	.002	.001
s20	-.014	-.007	-.005	-.004	-.005
s21	.264	.059	.012	.009	.014
s22	.063	.267	.025	.021	.026
s23	-.023	-.009	.024	-.002	-.003
s24	-.023	-.017	-.001	.047	-.003
s25	-.030	-.020	-.010	-.009	-.036
s26	-.038	-.026	-.010	-.007	-.011
s27	.004	.014	.012	.013	.013
s28	-.017	-.017	-.013	-.011	-.012
s29	-.015	-.009	-.009	-.009	-.011
s30	-.021	-.015	-.008	-.007	-.008

	i26	i27	i28	i29	i30
s2	-.008	-.008	-.008	-.008	-.009
s3	-.003	-.002	-.002	-.002	-.003
s4	.000	.001	.001	.000	-.000
s5	-.011	-.010	-.009	-.011	-.011
s6	-.006	-.005	-.005	-.006	-.007
s7	-.003	-.002	-.002	-.002	-.003
s8	-.014	-.014	-.015	-.014	-.014
s9	-.005	-.005	-.005	-.004	-.005
s10	-.001	.000	-.000	-.000	-.001
s11	-.003	-.002	-.002	-.002	-.003
s12	.002	.004	.004	.001	.001

s13	-.010	-.008	-.008	-.011	-.011
s14	-.008	-.005	-.004	-.007	-.007
s15	.001	.002	.002	.002	.001
s16	.007	.009	.009	.008	.008
s17	-.002	.001	.001	-.000	-.001
s18	-.006	-.004	-.005	-.005	-.006
s19	.000	.002	.002	.001	.001
s20	-.005	-.004	-.003	-.004	-.004
s21	.015	.012	.012	.011	.011
s22	.027	.024	.025	.023	.023
s23	-.003	-.003	-.002	-.003	-.004
s24	-.004	-.003	-.002	-.003	-.004
s25	-.012	-.009	-.009	-.010	-.011
s26	-.033	-.009	-.008	-.009	-.009
s27	.012	-.009	.012	.013	.013
s28	-.012	-.011	-.067	-.010	-.010
s29	-.010	-.009	-.009	.016	-.011
s30	-.008	-.007	-.007	-.007	.004

	sex2	race2	race3	race4	race5
s2	-.011	-.005	-.002	.010	.015
s3	-.032	-.003	-.001	-.004	.002
s4	.036	.021	.006	.001	.001
s5	.038	.016	.015	.010	.020
s6	.029	.007	.003	.009	.010
s7	-.020	-.019	-.010	.002	-.000
s8	-.001	-.010	.004	-.004	.000
s9	.009	-.011	-.009	.005	-.021
s10	.014	-.012	-.001	-.002	-.006
s11	.007	-.011	-.001	-.009	.005
s12	-.019	.019	.026	.002	.021
s13	-.014	.036	.022	.016	.022
s14	-.016	.029	.032	.010	.009
s15	.015	-.012	-.008	-.005	-.008
s16	.014	-.016	.001	-.002	-.005
s17	.025	-.010	.000	-.010	-.005
s18	.034	.013	.010	-.002	.008
s19	.002	-.019	.021	-.004	.024
s20	.017	-.003	.010	-.005	.011
s21	.008	.002	-.004	-.010	-.003
s22	.004	.013	.004	.004	-.001
s23	.007	.008	.005	.009	.007
s24	.017	.005	.010	.014	-.002

s25	-.003	-.002	.011	.006	.004
s26	.016	-.006	-.001	-.001	-.000
s27	.000	-.006	-.013	.010	.009
s28	-.008	-.014	-.019	-.010	-.000
s29	-.004	-.034	-.009	-.007	-.011
s30	.001	-.011	-.020	-.017	-.001

	race6	regi2	regi3	regi4	stoc2
s2	.012	.012	-.007	.007	.008
s3	.007	.018	.005	.008	-.013
s4	.009	.026	.015	.021	.011
s5	.009	.022	.008	.013	.005
s6	.008	.022	.012	.025	.003
s7	.003	.021	.022	.024	.005
s8	.003	.019	.025	.024	.010
s9	.014	.016	.003	.017	.003
s10	-.005	.022	.014	.021	-.008
s11	-.006	.026	.018	.025	-.002
s12	-.000	.030	.012	.023	.017
s13	.012	.026	.021	.029	.026
s14	.013	.026	.017	.029	.008
s15	.001	.016	-.003	.009	.000
s16	-.006	.023	.002	.012	.009
s17	-.008	.017	.013	.012	.004
s18	.020	.020	.008	.016	.003
s19	-.008	.026	.011	.013	.019
s20	-.012	.019	.017	.014	.011
s21	.003	.001	.008	.006	.008
s22	.002	-.007	.012	.003	.004
s23	.006	.025	.016	.020	.009
s24	.005	.029	.012	.016	.004
s25	.005	.017	.010	.023	.015
s26	.005	.022	.006	.025	.013
s27	-.000	.008	-.002	.006	-.012
s28	.006	.010	-.000	.009	.007
s29	.003	.009	.012	.015	.010
s30	.004	.003	.001	.011	.006

	stoc3	stoc4	stoc5	stoc6	stoc7
s2	-.001	.004	-.003	-.002	-.000
s3	-.007	-.007	-.012	-.014	-.006
s4	-.015	-.003	-.007	-.011	-.011

s5	-.003	.004	-.003	-.005	-.002
s6	-.016	-.000	-.010	-.008	-.008
s7	.001	.002	-.003	-.006	.010
s8	.002	.007	-.002	.003	.008
s9	-.009	-.004	-.010	-.005	-.003
s10	-.015	-.009	-.016	-.012	-.014
s11	-.013	-.001	-.011	-.009	-.007
s12	-.021	-.001	-.011	-.010	-.002
s13	-.008	.010	-.001	.009	.009
s14	-.010	-.003	-.013	-.002	.000
s15	-.011	-.003	-.019	-.008	-.003
s16	-.011	-.003	-.016	-.014	-.007
s17	-.017	-.004	-.012	-.016	-.006
s18	-.008	.002	-.011	-.008	-.004
s19	.006	.015	.008	.007	.005
s20	.006	.011	-.003	.003	.001
s21	-.003	.007	.006	.003	.006
s22	-.010	-.001	.006	-.003	.002
s23	-.011	-.001	-.004	-.001	.001
s24	-.007	.002	-.008	-.007	-.004
s25	.007	.011	.006	.006	.010
s26	-.008	.012	-.004	.001	.002
s27	-.011	-.011	-.005	-.012	-.014
s28	-.004	.007	-.003	-.001	.003
s29	.011	.011	.009	.010	.013
s30	.004	.003	-.008	.008	-.001

	pare2	pare3	pare4	pare5	pare6
s2	.007	.011	.007	.010	.008
s3	.004	.003	.000	.009	.007
s4	-.007	-.008	-.018	-.007	-.004
s5	.002	.001	-.003	.007	.004
s6	-.000	-.001	-.006	.002	.003
s7	-.004	-.005	-.009	-.002	-.000
s8	.005	.004	.001	.007	.009
s9	.001	.002	-.002	.005	.005
s10	-.004	-.002	-.007	.003	.003
s11	-.005	-.004	-.005	.003	-.001
s12	-.011	-.005	-.018	-.006	-.006
s13	-.004	-.004	-.014	-.006	-.003
s14	.001	-.004	-.011	-.001	-.000
s15	-.002	-.005	-.006	-.001	.002
s16	-.008	-.009	-.012	-.006	-.007

s17	.001	-.000	-.006	.002	.002
s18	-.001	.000	-.004	.001	.002
s19	-.012	-.013	-.019	-.011	-.012
s20	-.009	-.00	-.010	-.003	-.002
s21	-.019	-.016	-.023	-.021	-.016
s22	-.028	-.030	-.035	-.032	-.025
s23	-.005	-.007	-.010	-.002	-.002
s24	-.007	-.003	-.009	-.003	.000
s25	-.002	-.001	-.007	.002	.003
s26	-.003	.002	-.003	.005	.004
s27	-.014	-.011	-.012	-.012	-.008
s28	.008	.012	.011	.014	.010
s29	-.002	.003	.005	.007	.005
s30	.005	.003	.001	.008	.006

	s2	s3	s4	s5	s6
s2	1.0				
s3	.678	1.0			
s4	.632	.633	1.0		
s5	.623	.628	.670	1.0	
s6	.632	.647	.685	.652	1.0
s7	.646	.679	.651	.626	.647
s8	.521	.529	.524	.505	.539
s9	.643	.668	.647	.631	.653
s10	.663	.684	.670	.653	.674
s11	.673	.697	.676	.662	.678
s12	.698	.718	.704	.688	.710
s13	.673	.693	.686	.669	.684
s14	.699	.722	.706	.692	.710
s15	.547	.567	.553	.540	.556
s16	.652	.672	.656	.638	.658
s17	.595	.618	.607	.586	.609
s18	.622	.644	.630	.620	.630
s19	.601	.625	.604	.588	.601
s20	.585	.603	.594	.578	.606
s21	.649	.666	.661	.640	.657
s22	.654	.678	.670	.649	.668
s23	.692	.717	.699	.680	.700
s24	.669	.692	.679	.662	.679
s25	.642	.665	.649	.632	.653
s26	.652	.673	.658	.641	.663
s27	.134	.142	.142	.133	.144
s28	.269	.277	.274	.261	.274

s29	.493	.502	.498	.487	.508
s30	.532	.555	.538	.526	.532

	s7	s8	s9	s10	s11
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s7	1.0				
s8	.598	1.0			
s9	.663	.541	1.0		
s10	.679	.557	.686	1.0	
s11	.691	.565	.697	.711	1.0
s12	.716	.585	.688	.724	.725
s13	.688	.560	.666	.684	.694
s14	.713	.586	.696	.721	.728
s15	.562	.463	.572	.590	.597
s16	.674	.544	.672	.696	.699
s17	.618	.507	.618	.636	.645
s18	.633	.515	.644	.655	.666
s19	.607	.482	.602	.620	.630
s20	.599	.488	.590	.612	.619
s21	.672	.547	.656	.675	.684
s22	.680	.555	.665	.683	.692
s23	.709	.582	.694	.716	.729
s24	.686	.560	.678	.695	.706
s25	.669	.545	.652	.670	.679
s26	.671	.549	.661	.676	.688
s27	.139	.119	.138	.141	.143
s28	.269	.236	.273	.283	.279
s29	.516	.422	.506	.518	.527
s30	.548	.455	.548	.556	.566

	s12	s13	s14	s15	s16
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s12	1.0				
s13	.787	1.0			
s14	.798	.771	1.0		
s15	.588	.565	.593	1.0	
s16	.702	.669	.700	.577	1.0
s17	.644	.619	.649	.532	.658
s18	.671	.646	.673	.547	.664
s19	.653	.630	.652	.517	.615
s20	.640	.618	.638	.505	.600
s21	.712	.682	.706	.560	.665
s22	.721	.691	.719	.565	.674
s23	.759	.733	.761	.595	.703

s24	.731	.707	.732	.577	.682
s25	.700	.676	.707	.554	.660
s26	.712	.688	.718	.562	.669
s27	.146	.141	.148	.119	.137
s28	.287	.276	.289	.230	.273
s29	.538	.510	.540	.425	.508
s30	.571	.558	.577	.462	.549

	s17	s18	s19	s20	s21
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s17	1.0				
s18	.608	1.0			
s19	.558	.583	1.0		
s20	.549	.578	.579	1.0	
s21	.616	.638	.602	.593	1.0
s22	.624	.647	.607	.602	.742
s23	.651	.675	.646	.635	.695
s24	.632	.653	.634	.618	.671
s25	.610	.626	.609	.595	.644
s26	.617	.634	.617	.605	.643
s27	.127	.128	.125	.128	.137
s28	.251	.265	.250	.241	.270
s29	.473	.484	.461	.462	.503
s30	.506	.526	.502	.497	.535

	s22	s23	s24	s25	s26
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s22	1.0				
s23	.712	1.0			
s24	.679	.740	1.0		
s25	.657	.710	.690	1.0	
s26	.658	.710	.707	.684	1.0
s27	.150	.145	.151	.139	.162
s28	.270	.287	.285	.273	.276
s29	.510	.539	.524	.514	.508
s30	.543	.579	.564	.540	.550

	s27	s28	s29	s30
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s27	1.0			
s28	.074	1.0		
s29	.104	.205	1.0	
s30	.113	.239	.367	1.0
