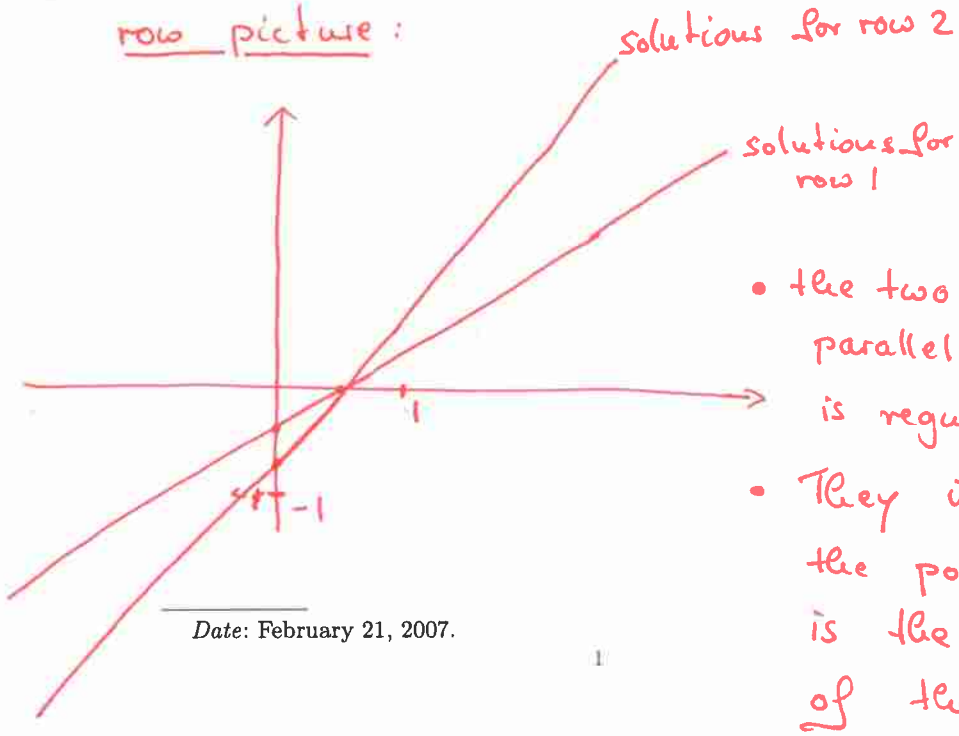
225 MIDTERM 1, SPRING 2007

(1) **Make sure to do all parts!** Write the following systems of equations in matrix form. For both of them, draw the row picture and the column picture and explain (with both pictures) whether the matrix is singular or regular, and whether the system has zero, one or infinitely many solutions. Then determine the set of solutions.

(a)

$$\begin{aligned} 2x + -3y &= 1 \\ -8x + 6y &= -4 \end{aligned} \qquad \begin{pmatrix} 2 & -3 \\ -8 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

row picture:

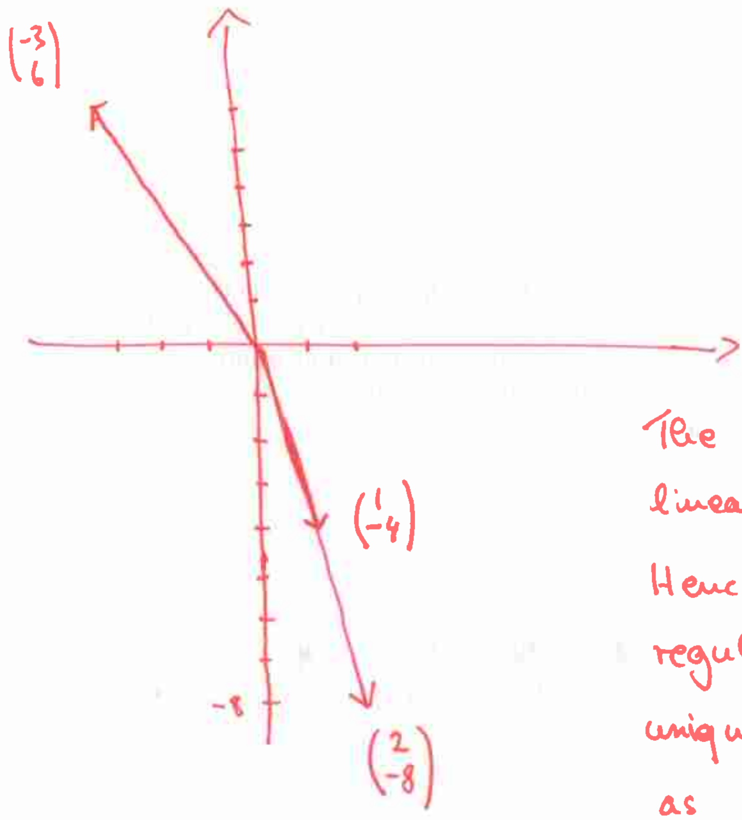


- the two lines are not parallel <=> the matrix is regular.
- They intersect in the point $\begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$, which is the unique solution of the system.

Date: February 21, 2007.

column picture (for (a))

$$x\begin{pmatrix}2\\-8\end{pmatrix}+y\begin{pmatrix}-3\\6\end{pmatrix}=\begin{pmatrix}1\\-4\end{pmatrix}$$



The two column vectors are linearly independent.

Hence the matrix is regular, and there is a unique way to express $\begin{pmatrix}1\\-4\end{pmatrix}$ as a linear combination of $\begin{pmatrix}-3\\6\end{pmatrix}$ and $\begin{pmatrix}2\\-8\end{pmatrix}$, namely:

$$\begin{pmatrix}1\\-4\end{pmatrix}=\frac{1}{2}\cdot\begin{pmatrix}2\\-8\end{pmatrix}+\underbrace{0\cdot\begin{pmatrix}-3\\6\end{pmatrix}}_{\text{(it is ok not to write this part.)}}.$$

(b)

$$\begin{aligned} 2x + -3y &= 3 \\ -8x + 7y &= 7 \end{aligned}$$

similar to (a)

(2) Write the following system of equation in matrix form:

$$\begin{aligned} x + y - 2z &= 4 \\ -x + 3y + 2z &= 1 \\ 2x + 11y - 3z &= 5. \end{aligned}$$

$$\begin{pmatrix} 1 & 1 & -2 \\ -1 & 3 & 2 \\ 2 & 11 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}$$

Then write down the augmented matrix $[A|b]$. Use Gauss' algorithm to solve the system (in the augmented matrix form).

$$\begin{array}{r} \\ R1+R2 \to \\ R3-2R1 \to \end{array}\left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ -1 & 3 & 2 & 1 \\ 2 & 11 & -3 & 5 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 4 & 0 & 5 \\ 0 & 9 & -7 & -3 \end{array}\right] \leftarrow R_3 - \frac{9}{4} R_2$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 4 & 0 & 5 \\ 0 & 0 & -7 & -\frac{57}{4} \end{array}\right] \begin{array}{l} \\ \leftarrow \cdot \frac{1}{4} \\ \leftarrow \cdot \frac{-1}{7} \end{array} \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 4 & 0 & 5/4 \\ 0 & 0 & 1 & -\frac{57}{28} \end{array}\right] \begin{array}{l} \leftarrow R_1 - R_2 + 2R_3 \\ \\ \\ \end{array}$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{37}{28} \\ 0 & 1 & 0 & 5/4 \\ 0 & 0 & 1 & -\frac{57}{28} \end{array}\right]$$

check: $$\begin{pmatrix} 1 & 1 & -2 \\ -1 & 3 & 2 \\ 2 & 11 & -3 \end{pmatrix} \begin{pmatrix} -\frac{37}{28} \\ 5/4 \\ -\frac{57}{28} \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix} \checkmark$$