(3) (a) How many solutions does the system of question (2) have?

one

(b) What does that tell you about the rank of the matrix A?

A has full rank (rank 3)

(c) Is A invertible?

yes, because it has full rank

(d) What is the determinant of A?

$1 \cdot 4 \cdot (-7) = -28$

(e) If you would replace b with a different vector, how many solutions do you expect the system to have?

still one, since A is regular

(f) What is the dimension of the column space of A?

it is 3 (= rank A)

(4) Let A be an 3×3 matrix of rank r. True or false:

(a) The rank r of A equals the dimension of the solution space of $Ax = b$.

false

(b) For any $b \in \mathbb{R}^n$, the dimension of the solution space of $Ax = b$ equals $3 - r$.

false

(c) The rank r of A equals the dimension of the solution space of $Ax = 0$.

false

(d) The dimension of the solution space of $Ax = 0$ equals $3 - r$.

true