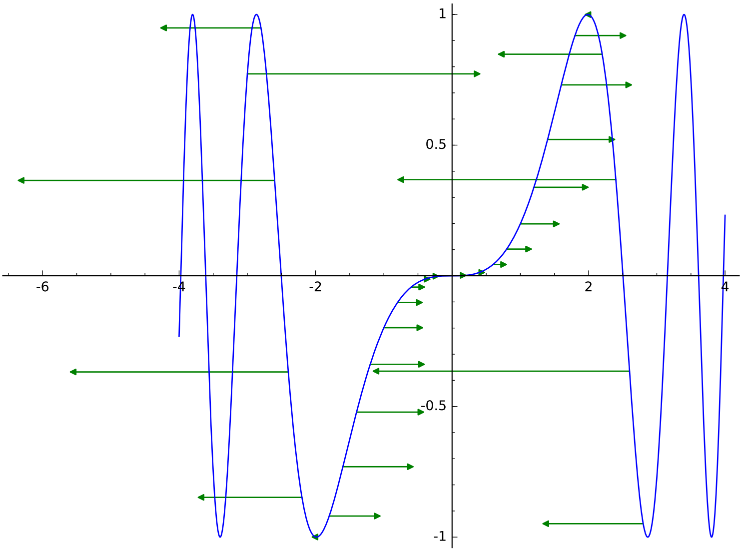
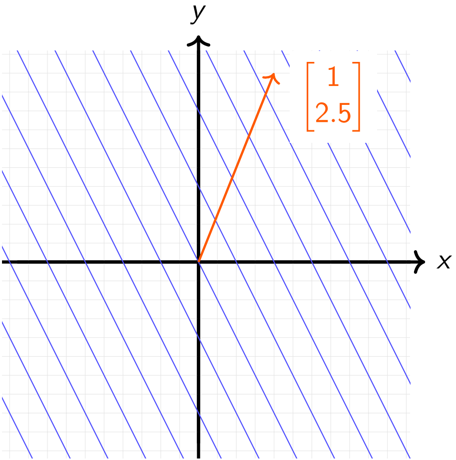
Gradior
Latin:
stride
(x0,y0) you are here
Peak
gradient walk
Level Scenic Round
Peak
∇f
Level Scenic Round
elevation f(x0,y0)

Gradient of a function of one variable



Linear functions in two variables



The slanted plane is determined by its height at one point

$$f(x_0, y_0) = z_0$$

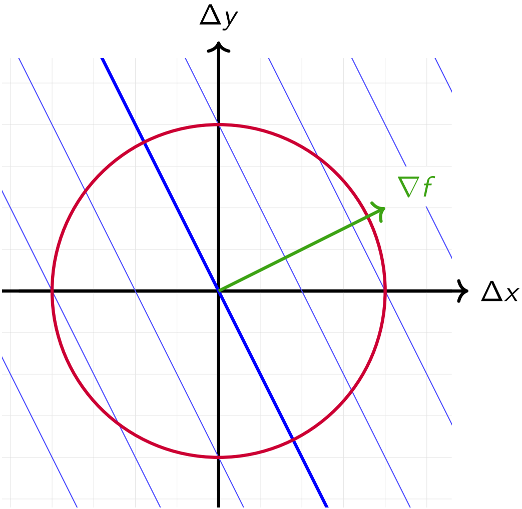
along with its slopes in two directions, typically,

$$f_x = \frac{\Delta f}{\Delta x} \quad \text{and} \quad f_y = \frac{\Delta f}{\Delta y}$$

$f(1,0) - f(0,0) = f_x$ and $f(1,2.5) - f(1,0) = 2.5 \cdot f_y$, so

$f(1,2.5) - f(0,0) = 1 \cdot f_x + 2.5 \cdot f_y$

Walking around the unit circle: directional derivatives



Slope in $\hat{\boldsymbol{u}}$-direction

$\nabla f \bullet \hat{\boldsymbol{u}}$

Position at time t

$$\hat{\boldsymbol{u}}(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$$

Height at time t

$$z_0 + f_x \cos(t) + f_y \sin(t)$$

Elevation above z_0

$$\begin{bmatrix} f_x & f_y \end{bmatrix} \circ \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$$

Which direction is steepest? Which is level?

Elevation (above z_0) is zero:

$$0 = \nabla f \bullet \hat{\boldsymbol{u}}(t) = f_x \cdot \cos(t) + f_y \cdot \sin(t)$$

The level line is perpendicular to the gradient.

Critical points:

$$0 = \nabla f \bullet \hat{\boldsymbol{u}}'(t) = -f_x \cdot \sin(t) + f_y \cdot \cos(t)$$

$$\hat{\boldsymbol{u}}_{max} = \frac{\nabla f}{|\nabla f|} \qquad \hat{\boldsymbol{u}}_{min} = -\frac{\nabla f}{|\nabla f|}$$

The gradient is pointing into the steepest direction.

The length of the gradient is the slope in this steepest direction.