Topological order in ground state wave functions of gapped spin chains with continuous symmetry

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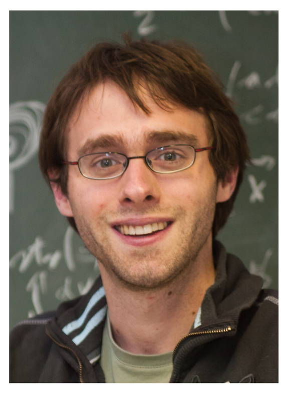
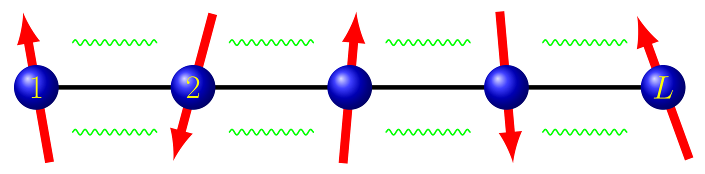



Physical setup

Quantum system of spins $\vec{S}_l$ transforming in a Lie algebra $\mathfrak{g}$

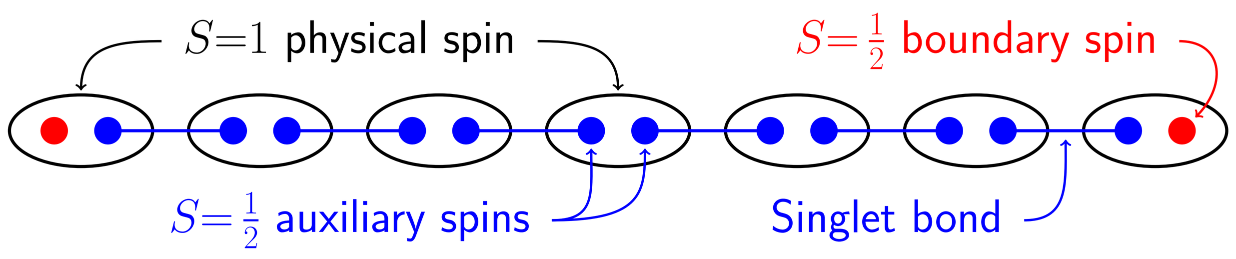


$\rightarrow\ |\psi\rangle \in \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_L$

Assumptions:
- $\mathcal{H}_l$ are irreps of $\mathfrak{g}$
- Hamiltonian H is $\mathfrak{g}$-invariant
- Unique ground state $|\psi\rangle$
- Energy gap (will constrain choice of irreps $\mathcal{H}_l$)

The Haldane phase of $\mathrm{SU}(2)$ spin chains

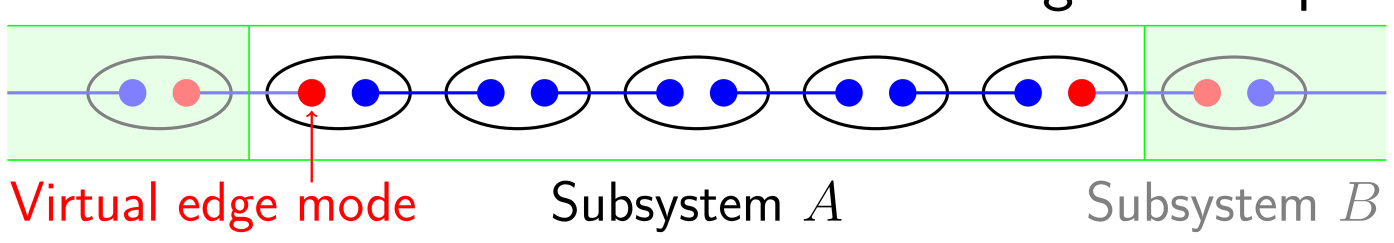
The open $S{=}1$ chain exhibits gapless $S{=}\frac{1}{2}$ boundary spins:



Explicit realization by AKLT in terms of an MPS/VBS state

Symmetry fractionalization:
Center $\mathbb{Z}_2 \subset SU(2)$ acts trivially on physical spins...
...but non-trivially on emergent boundary spins

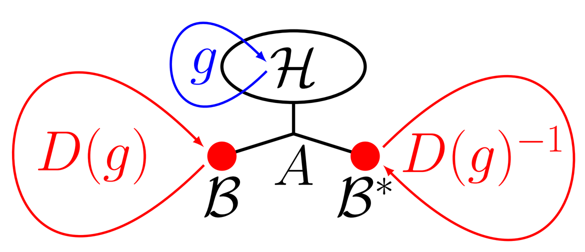
Virtual realization of boundaries in the entanglement spectrum:



Classification of topological phases

Idea: Understand the action of the symmetry group $\mathcal{G}$ on the boundary spins or on the entanglement spectrum [1, 2].

Topological phases $\Leftrightarrow$ Classes of projective reps of $\mathcal{G}$



$\mathcal{G}$-intertwiner $A:\ \mathcal{B}\otimes\mathcal{B}^* \rightarrow \mathcal{H}$ lifts symmetry from auxiliary to physical level

Boundary reps $\mathcal{B}$ and $\mathcal{B}^*$ only need to be *projective* reps of $\mathcal{G}$:

$$D(g_1)D(g_2) = e^{i\omega(g_1,g_2)}D(g_1g_2)$$

Some basic facts on simple Lie groups

All Lie groups with Lie algebra $\mathfrak{g}$ can be written as G/Γ with
- G simply-connected (no holes, $\pi_1(G) \cong \{1\}$)
- $\Gamma \subset \mathcal{Z}(G)$ subgroup of the center
- Projective representation of G_Γ = linear representation of G

For the projective group $PG = G/\mathcal{Z}(G)$:
- Projective class of G-irrep $\lambda \Leftrightarrow$ irrep $[\lambda]$ of center $\mathcal{Z}(G)$
- Number of topological phases: $|\mathcal{Z}(G)|$
- Identity: $\mathcal{Z}(G) \cong$ Irreps of $\mathcal{Z}(G) \cong$ weight/root lattice of $\mathfrak{g}$

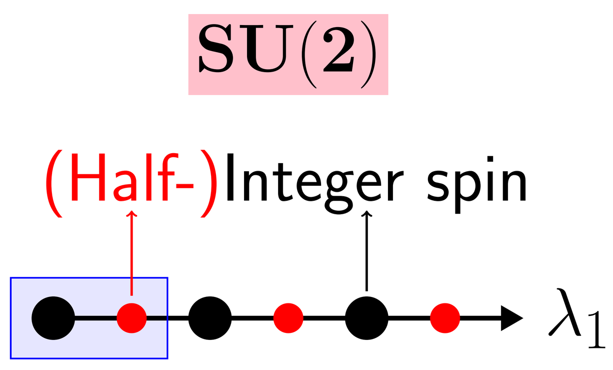
From boundary reps $\mathcal{B}$ to topological classes

For the center of $SU(N)$ one immediately finds

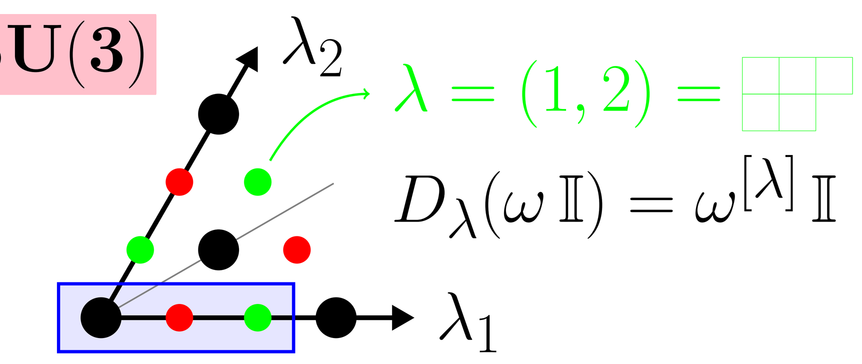
$$\mathcal{Z}\big(SU(N)\big) \cong \mathbb{Z}_N \cong \big\{\mathbb{I}, \omega\,\mathbb{I}, \cdots, \omega^{N-1}\mathbb{I}\big\} \quad \text{with} \quad \omega = e^{\frac{2\pi i}{N}}$$

PSU(N) spin chains admit N distinct phases

Type	Label λ	Topological class $[\lambda]$
Dynkin label	$(\lambda_1,\ldots,\lambda_{N-1})$	$\sum k\lambda_k \mod N$
Young tableau		Boxes$(\lambda) \mod N$



- Linear reps of $SO(3)$
- Projective reps



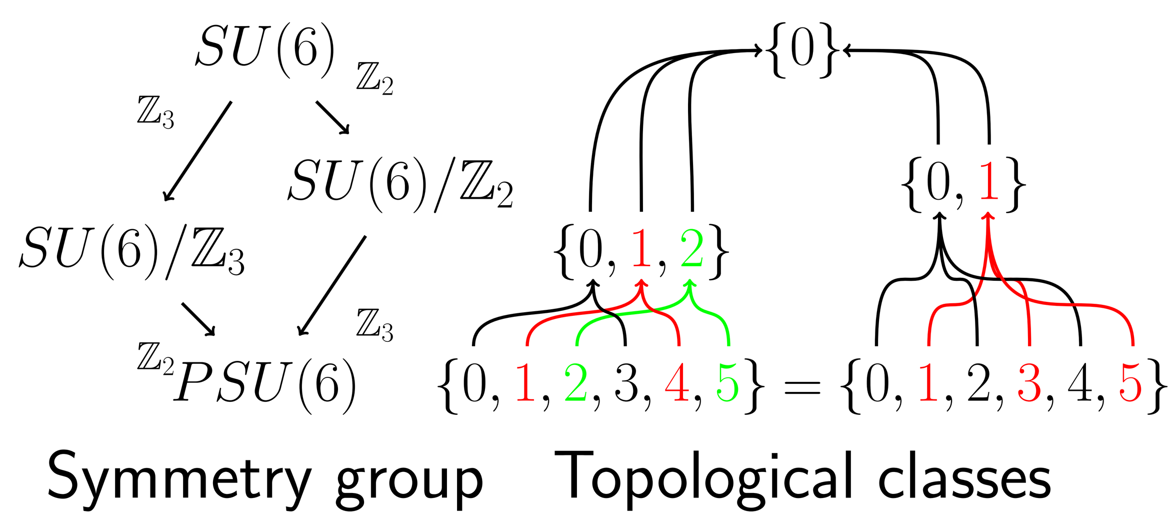
- Linear reps of $PSU(3)$
- Projective reps

Results for all classical Lie groups

For symmetries of type $su(N)$ and $so(2N)$ there exists more than one topologically non-trivial phase

$\mathfrak{g}$	$su(N)$	$so(2N{+}1)$	$sp(2N)$	$so(4N{+}2)$	$so(4N)$
G	$SU(N)$	$Spin(2N{+}1)$	$Sp(2N)$	$Spin(4N{+}2)$	$Spin(4N)$
$\mathcal{Z}(G)$	$\mathbb{Z}_N$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_2$

Hierarchies of topological phases



Different choices of symmetry group lead to different sets of topological phases

$\Downarrow$

New perspective on Haldane's Conjecture

Outlook

- We constructed a string order parameter which can distinguish the N distinct phases of $PSU(N)$ spin chains, see [6].
- Inversion symmetry requires $\mathcal{B} \cong \mathcal{B}^* \Rightarrow$ restriction $2[\lambda] \equiv 0$
- Realization in terms of cold atoms? ($\rightarrow$ [7])
- Classification of supersymmetric and q-deformed spin chains

Literature

[1] X. Chen, Z.-C. Gu, and X.-G. Wen, *Phys. Rev.* B83 (2011) 035107.
[2] N. Schuch, D. Pérez-García, and I. Cirac, *Phys. Rev.* B84 (2011) 165139.
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[5] K. Duivenvoorden and T. Quella, arXiv:1206.2462.
[6] K. Duivenvoorden and T. Quella, arXiv:1208.0697.
[7] H. Nonne, M. Moliner, S. Capponi, P. Lecheminant and K. Totsuka, arXiv:1210.2072.