Preface

This is the proceedings of a concentration month devoted to “Quantum symmetries” that took place at the Centre de Recherches Mathématiques, Université de Montréal, from October 10 to November 4, 2022. It was one of six such months that constituted the Thematic Semester entitled “Symmetries: Algebras and Physics”. Our month featured a one week international conference, three weeks of lecture courses (some introductory and some advanced), problem sessions and additional research seminars. We thank our coorganisers Anne Moreau and Kolya Reshetikhin for their assistance with putting together this program.

Our interpretation of “Quantum symmetries” is that it serves as a common language to connect many important fields of modern mathematics and mathematical physics. This language is that of tensor categories. These mathematical structures were introduced by MacLane and Bénabou in the 1960s and was invigorated in the 1980s by the works of Drinfeld, Jones, Moore and Seiberg, Reshetikhin and Turaev, Verlinde and Witten, among others.

Classically, the symmetries of an object are encoded by a group. This paradigm changed rapidly in both mathematics and physics with the emergence of, and subsequent advances in, fields such as conformal field theory (vertex algebras) and integrability (quantum groups). In their simplest appearances, quantum symmetries arise when considering the space of states of a physical system, typically built from a braided tensor category of modules over some symmetry group or algebra. They moreover imbue the category with beautiful additional features, leading one to the study of what are now called spherical, fusion and modular categories (to name a few). These structures generalise the notion of symmetry far beyond that of a group action.

The particular foci of our concentration month included the theory of abstract tensor categories (*eg.*, fusion, modular and non-semisimple generalisations), as well as applications to Hopf algebras, vertex algebras, invariants of links and 3-manifolds, subfactors, topological quantum field theories and (of course) physics. This theory and its myriad of applications are now absolutely essential parts of the modern mathematician’s and mathematical physicist’s toolkit. We hope that this collection of reviews and articles will not only be valuable to these researchers, but also inspire them to build even further on the results obtained to date.

We conclude with a brief overview of the articles that appear in this proceedings.

Jethro van Ekeren reviews W-algebras, which are constructed from affine vertex algebras via quantum Hamiltonian reduction associated to a nilpotent element inside the underlying Lie algebra. Depending on the nilpotent element, there are exceptional levels for which the simple W-algebra is strongly rational. In these cases, he also discusses the modular tensor categories formed by their representations.

Justine Fasquel gives explicit formulae for the operator product algebra of W-algebras associated to Lie algebras of rank two. Such formulae are in general hard to obtain and they can (and are) used to find exceptional isomorphisms between vertex algebras.

Andrew Riesen takes the first steps toward an understanding of hypergroup actions on vertex operator algebras. He studies this in the case of strongly rational vertex operator algebras and, in particular, shows that any strongly rational vertex operator subalgebra can be characterized as the fixed points of a hypergroup. These findings are illustrated for the exceptional extensions of the rational affine vertex algebras of $\mathfrak{sl}_2$.

Alissa Furet and Theo Johnson-Freyd consider ground states in twisted sectors of the Conway moonshine holomorphic superconformal field theory. This vertex operator superalgebra has the Conway group, a sporadic group, acting as automorphisms. Accordingly, there is one twisted sector for each conjugacy class and they examine if each sector possesses a single parity or if it has both even and odd states.

Simon Lentner studies a combinatorial problem for finite abelian groups that is motivated by classifying R-matrices for quantum groups.

Modular data is expected to be realized by some category. **Eric C. Rowell, Hannah Solomon and Qing Zhang** accomplish this for some previously unrealized super-modular data by modifying the Drinfeld centers of near-group fusion categories associated with certain abelian groups.

Andrew Schopieray studies fusion ring automorphisms, leaving only the trivial element fixed. He proves that a variety of classical results on fixed-point-free automorphisms of finite groups are true in the generality of fusion rings.

Theo Johnson-Freyd gives proofs of the classification of $(3+1)D$ topological orders with only a $\mathbb{Z}_2$-charged particle, both in the even and odd case.

Jürgen Fuchs, Gregor Schaumann, Christoph Schweigert and Simon Wood discuss under which circumstances the distributors between the two tensor products in Grothendieck–Verdier categories are isomorphisms.

Christian Blanchet, Martin Palmer and Awais Shaukat study the action of subgroups of the mapping class group on Heisenberg homologies.

Daniel Berwick-Evans, Emily Cliff, Laura Murray, Apurva Nakade and Emma Phillips construct, for a weak 2-group $\mathcal{G}$, a bicategory of flat $\mathcal{G}$-bundles over differentiable stacks as a localization of a functor bicategory.

Jin-Cheng Guu is computing the Crane-Yetter theory in (co)dimension 2. Due to the similarity with the Drinfeld center, he calls this the categorical center of higher genera.

Joseph Vulakh makes progress towards a general classification of twisted homogeneous racks of type D by proving that several families of twisted homogeneous racks arising from alternating groups are of type D.