PART E: PROPOSAL DESCRIPTION

E1: Proposal Title

Indecomposable Structure in Representation Theory and Logarithmic Conformal Field Theory.

E2: Aims and Background

The research which I propose to undertake may be classed under the broad umbrella of conformal field theory (CFT) [1][1]. CFTs are quantum field theories which are not just invariant under the Lorentz group of special relativity, but are in fact invariant under a larger group of symmetries, known as the conformal group. The study of conformal invariance takes on a special flavour in two dimensions, because in this case the corresponding algebra of infinitesimal symmetries is infinite-dimensional, leading to a substantial increase in our control, and understanding, of the field theory. Indeed, in many cases the theory can be in principle exactly solved. CFT without qualification generally refers to this two-dimensional case.

Despite the seemingly special restrictions of a two-dimensional theory, CFT is an extremely active area of research in theoretical and mathematical physics. One reason for this is that CFT is at the heart of modern researches into statistical mechanics and string theory. The former because of the fact that statistical models display conformal invariance at their critical points, and the latter because a string theory must be conformally invariant when "pulled back" to the two-dimensional "worldsheet" swept out in time by the one-dimensional string.

The most surprising application of CFT is, beyond doubt, to pure mathematics. CFT and its extension, string theory, have played an enormous role in shaping and unifying modern mathematics in recent years. Indeed, the concept of a vertex operator algebra, originally developed as a framework in which to prove the famous "monstrous moonshine" conjectures, is nothing more than the mathematician's refinement (to their impeccable standards of rigour) of a (chiral) CFT. The role of CFT in relating diverse mathematical subjects such as infinite-dimensional algebras, stochastic Loewner evolution (SLE), knot theory, combinatorics, number theory, quantum groups, and algebraic geometry (to name but a few) is well-known. Indeed, the awards list of the most coveted prize in mathematics, the Fields Medal, turns up Jones and Witten (knot theory), Borcherds (vertex algebras) and Werner (SLE). I think it fair to say that CFT has been one of the most important developments for mathematics during the last quarter of a century.

The focus of the research proposed here is on a generalisation of CFT which has come to be known through the qualifier "logarithmic". Logarithmic CFTs were first discussed in the early nineties [2, 3] and are so named because they admit correlation functions, vacuum expectation values of products of quantum fields, with logarithmic branch cuts. This contrasts with standard (rational) CFT in which the branch points of the correlation functions are of finite order (power-type).

Despite a promising beginning, logarithmic CFT quickly attained a reputation for being esoteric and technical. Some impressive examples were constructed, in particular the triplet model [4–6], but the field suffered from a perceived lack of concrete applications. To be sure, there were many attempts to use logarithmic theories to explain discrepancies in models of the fractional quantum Hall effect, abelian sandpiles, D-brane recoil and more (see [7] for references to these), but none of these attempts really left an enduring mark upon their intended field. One possible reason for this is that logarithmic CFTs are inherently non-unitary, and so could be easily disregarded as unphysical.

[1] Citations here refer to section E8. Space restrictions unfortunately mean that these references cannot be indicative of the entire field and I apologise for the many important articles which have been omitted.

Nevertheless, condensed matter physicists remained interested in these theories for the simple reason that the standard CFT description of their favourite models was known to be incomplete. CFT is celebrated throughout this field for providing a beautiful description of *local* statistical observables in which expectation values and critical exponents may be deduced from CFT correlation functions. For example, the local observables of the Ising model correspond to the primary fields of the minimal model CFT $\mathcal{M}(3,4)$. However, there are many models in which the observables of interest are not local, and here standard CFT appears to have very little to say.

The archetypal example of such a statistical model is percolation. Percolation itself refers to a collection of closely related problems in probability theory, but these problems exhibit critical behaviour analogous to that of phase transitions in macroscopic media [8]. The interest from the physics community then stems from regarding percolation as a collection of relatively simple models with which one can test predictions such as conformal invariance at criticality and universality. The setup is as follows: One considers a fixed rectangular subdomain of a square lattice, and considers random configurations in which each edge of the lattice is chosen to be open or closed with probability p or $1-p$. The basic question of percolation is then to determine the probability that such a random configuration will contain a cluster of edges connecting one of the vertical sides of the rectangle to the other. In the continuum limit where the lattice spacing tends to zero, this crossing probability is only interesting (neither zero nor one) for a critical value of p. The fundamental observable of percolation is then the crossing probability for this critical p. As this requires a global knowledge of each random configuration, the crossing probability is referred to as a *non-local* observable.

Simple arguments suggest that the CFT describing critical percolation must have a vanishing "central charge". But standard CFTs with this property are completely trivial and cannot describe any (interesting) physical model. Nevertheless, one of the most celebrated confirmations of conformal invariance in condensed matter physics is the derivation, due to John Cardy [9], of an exact formula for the crossing probability in percolation. His derivation assumes that the limit is conformally invariant, and then relies only upon standard CFT techniques. It should be stressed that his article says little about the CFT itself. However, the agreement of his formula with the numerical simulations of [10] is very impressive.

In collaboration with Pierre Mathieu, I have proven that Cardy's derivation of the percolation crossing probability defines a CFT which we have verified to be logarithmic [11]. This comes amid a recent resurgence in activity within the logarithmic CFT community, aimed at clarifying applications to theoretical physics and developing the mathematical properties of logarithmic theories so as to more closely mirror those of standard CFTs. One can isolate several different approaches including free field methods and quantum group connections [12–14], lattice model constructions [15–18] and construction through explicit fusion [19–21]. At around the same time, high-energy physicists started to reconsider models of string theories in which the target space is a Lie supergroup [22–24]. Whilst these models have a long history in string theory, they cannot be said to be nearly as well-understood as their bosonic analogues, despite the role they play in important topics such as the AdS/CFT correspondence. This attention has revived the realisation that these models are also logarithmic CFTs, and deserve to be understood as such. A third strand of interest has also arisen in the mathematical community, where efforts are underway to formally define appropriate notions of a logarithmic vertex operator algebra [25].

The core aim of the programme which I intend to follow during the course of this fellowship may be summarised as follows:

> **This project aims to study the mathematical structures underlying logarithmic conformal field theory, with a view to explore, unify and enable computation. Expected outcomes include the generation of a consistent and detailed formalism which naturally accommodates applications to condensed matter physics, string theory and mathematics.**

I shall detail more specific aims and outcomes in the technical section E4, where I shall also explain the relevance of "indecomposable structure in representation theory" (section E1). These specific aims take the form of sub-projects, each of which is rooted in articles I have published in this field

[11, 20, 21, 26] and current work in progress. For further background of a more technical nature, the reader is referred to the descriptions of these sub-projects.

E3: Significance and Innovation

Logarithmic CFT has a relatively short history — only fifteen years have elapsed since the original papers on the subject. But, as mentioned above, the last few years have seen a substantial improvement in our understanding of this seemingly technical subject. This is a rapidly emerging area of mathematical physics which is currently attracting more and more interest internationally, both for the sake of fundamental research and with a view to potential applications. Australia is already contributing strongly to this fundamental research effort and the project outlined here aims to significantly boost and complement our current strengths. We are well-poised to situate ourselves as the leaders in the field and should make every effort to capitalise on this chance.

As noted, one motivating reason to develop the theory of logarithmic CFTs is that they describe the statistical behaviour of non-local observables in critical condensed matter systems. Besides the example of percolation (section E2), such systems include mathematical idealisations of solutions of polymers, both dilute (low concentration) and dense (high concentration), as well as a wealth of other important models. The value of a complete description of the observables of these systems, both for fundamental and applied science, should be evident. Indeed, the value of the corresponding descriptions for local observables in the Ising, tricritical Ising, 3-state Potts, etc... models is well documented (see [1, Chs. 7.4 and 12] for an overview).

Besides this application to condensed matter theory and all that such applications entail, logarithmic CFT also has a relatively under-exploited role to play in modern string theory research, in particular to theories with supersymmetric backgrounds. Indeed there are even recent claims that logarithmic structure in such theories is a gravitational effect [27]. Examples of such backgrounds include Lie supergroups of PSU type and their symmetric spaces. The corresponding string theories are prototypes for the famous AdS/CFT correspondence of Maldacena [28] (see section E4, project 6). In addition, the mathematical structures underlying logarithmic CFT have recently been proven [29] to be present in stochastic Loewner evolution (SLE). SLE is a rigorous probabilistic approach to the models of condensed matter physics which has already been hailed as one of the great mathematical breakthroughs of recent times. Structural results from logarithmic CFT will therefore be of great interest to this mathematical community, and work is already underway to clarify and extend the relationship between SLE and logarithmic CFT.

One key point of this proposal is that to obtain a useful mathematical framework for these applications, we will have to devise methods to enable us to actually compute the quantities which are required. I will argue that the current state of knowledge is not sufficiently mature to accommodate this natural desire. Not only are we arduously exploring a class of (presumably physical) logarithmic CFTs, but we are doing so without much concern for explicit computation. Below, I outline several specific issues (and solutions) addressing this latter concern (section E4, projects 2 and 5) as well as a more general vision encompassing new classes of fundamental logarithmic CFTs whose elucidation is vital for improving upon our current knowledge.

E4: Approach and Methodology

Below I will outline in detail the sub-projects which I have in mind to fulfil the core aim of section E2, indicating my proposed approaches and methods. These are by no means exhaustive, but serve to indicate that there is a wealth of valuable work with which I will occupy myself, upon receiving funding. First however, it is necessary to introduce some of the technical concepts and nomenclature that are necessary for informed discussion.

The most important mathematical structure associated with a CFT, logarithmic or not, is its chiral symmetry algebra. This is a graded, infinite-dimensional associative algebra whose representations comprise the quantum state space of the theory. A chiral algebra may be a Lie algebra (the Virasoro algebra and affine Kac-Moody algebras are archetypal examples), a Lie superalgebra, or something more general (parafermionic algebras, W-algebras, ...). To characterise the CFT, it is therefore vital to understand the representation theory of the appropriate chiral algebra.

Concerning logarithmic CFTs, it was Gurarie [3] who realised that logarithmic branch cuts in correlation functions were an indication that a "zero-mode" of the chiral algebra, typically the Virasoro zero-mode L_0, can not be diagonalised but rather possesses a rank-2 Jordan cell. The zero-modes of a chiral algebra always commute with one another and their eigenvalues are typically interpreted as the quantum numbers which distinguish the states of the theory. Gurarie's insight was then that logarithmic CFTs are distinguished structurally from standard CFTs in having zero-modes with *generalised* eigenspaces.

This insight was confirmed rather dramatically by Gaberdiel and Kausch [4] who were able to explicitly construct representations on which L_0 was not diagonalisable using ideas of Gaberdiel [30] and Nahm [31]. Standard CFT presupposes that all representations appearing in the theory are completely reducible (can be expressed as direct sums of irreducible representations) and highest weight. However, when L_0 (or another zero-mode) is not diagonalisable, the representation is necessarily reducible but not completely so. Moreover, it need no longer be highest weight. Instead one speaks of direct sums of *indecomposable* representations generalising their highest weight cousins, hence the paradigm that logarithmic CFT roughly equates to indecomposable structure in representation theory[2].

With this background, the following research projects illustrate the directions I propose to pursue. Space constraints limit the number which can be adequately described, and there are many others which may prove fruitful. For example, I would also like to study integrable perturbations of logarithmic CFTs with a view to determining what happens to the quantum group symmetry of the standard case. It would moreover be interesting to further develop the bridge between logarithmic CFT and SLE. Finally, engaging in research typically leads to further directions of study and applications that one had not envisaged at the start. In any case, it is clear that there is much to be done!

1. Classification of Staggered Representations

The indecomposable representations relevant to logarithmic CFT were called *staggered* by Rohsiepe [32], who made significant progress in classifying simple examples of these representations. With Kalle Kytölä (Université de Genève), I have been generalising and extending Rohsiepe's results, and we have obtained a classification of the simplest class of staggered representations of the Virasoro algebra (those which are extensions of one highest weight representation by another with L_0 possessing Jordan cells of rank 2). This work is anticipated to appear on the arXiv in March or April.

Whilst the Virasoro representations we have classified cover the majority of those known to arise naturally in logarithmic CFT, they are not exhaustive. Extensions constructed from three highest weight representations have appeared in critical percolation [21], and representations with rank 3 Jordan cells for L_0 have also been constructed [19]. An obvious line of research is then to generalise our current results to cover these cases and more. An (ambitious) aim would be to classify all Virasoro staggered representations.

This is clearly a non-trivial exercise in pure mathematics (Lie theory and homological algebra), but its resolution is not only of mathematical importance, but is also critical to improving our understanding of logarithmic CFT. Unfortunately, constructing higher-rank staggered representations directly from lattice model techniques or fusing simpler representations is computationally intensive, and our algorithms are already reaching the limit of what is feasible. Progress in understanding what is physically possible therefore requires mathematical input to reduce the number of possibilities to something more manageable. Such progress will also be important to other related fields, in particular to the study of algebraic representations in SLE.

Along similar lines, the other projects proposed here will benefit enormously from similar classifications for other chiral algebras. A second branch to this project that I intend to explore concerns staggered representations for the affine Kac-Moody algebra $\widehat{\mathfrak{sl}}(2)$, as this case is expected to be the

[2]Strictly speaking, indecomposable structure is not enough, though the paradigm is often stated in this form. What is needed is indecomposable structure on which a zero-mode acts non-diagonalisably.

most tractable (the analysis of Kac-Moody symmetries is typically more straight-forward than that of Virasoro symmetries) and useful (see sub-projects 3 and 4 for example). However, there is significant interest in considering similar classifications for super-Virasoro algebras, simple Lie superalgebras and affine Kac-Moody superalgebras.

2. Free-Field Realisations and the Logarithmic Coupling

The staggered representations which arise most frequently in explorations of logarithmic CFT are generically described by a parameter β called the logarithmic coupling (or less imaginatively, the beta-invariant). This is most simply defined as the scalar product of the (normalised) zero-mode eigenstate and its Jordan cell partner [11], and its knowledge is necessary to characterise the mathematical action of the positive chiral algebra modes on the full representation. Indeed, its value is generally deduced from this action. Moreover, correlation functions involving Jordan cell partner fields generally involve the logarithmic coupling, so it is of direct physical interest as well. It follows from this that physical applications of any logarithmic CFT *require* knowledge of the appropriate logarithmic couplings.

This said, the logarithmic coupling is generally quite difficult to compute. Lattice model approaches have so-far proved fruitless as there one only obtains information about the Virasoro mode L_0 and not the positive modes needed for β. Explicit constructions of staggered representations via fusion allows the computation of β but this is computationally prohibitive except in the simplest cases. For certain representations, one may obtain β as a by-product of computing singular vectors [20], but this is again computationally intensive and not completely general.

To reiterate, computation within staggered representations requires the logarithmic coupling β as do the correlation functions of any physical application of the corresponding logarithmic CFT. It is therefore vital to develop better methods to compute these elusive invariants, and this is of course the aim of this project. In support of this aim, Pierre Mathieu and I have already noted [20] that the logarithmic couplings of certain examples of Virasoro representations can be expressed as simple functions of the central charge (they occur quite generally), suggesting that there are universal formulae for at least some classes of staggered representations.

The technical tool with which I propose to attack this computation is the formalism of free-field methods. This will be familiar to CFT researchers (see [1, Ch. 9] or [33]), and allows one to replace a chiral algebra with another for which computations are far easier. At the end, one performs a BRST-like projection (takes the cohomology of the free-field resolutions) to recover the required result for the original chiral algebra. Free-field approaches to logarithmic CFT have already been pioneered [12–14] with some success, but applications to computing β have not yet been considered.

3. Logarithmic CFTs with $\widehat{\mathfrak{sl}}(2)$ Symmetry

It is widely believed that every standard (rational) CFT can be related to Wess-Zumino-Witten (WZW) models by a few well-understood operations including orbifold and coset constructions. For example, the unitary minimal models may be uniformly constructed as certain cosets of WZW theories with $\widehat{\mathfrak{sl}}(2)$ symmetry. These WZW models are rational CFTs describing strings propagating on a compact Lie group, and are often referred to as the fundamental building blocks of rational CFT. It is therefore reasonable to expect that logarithmic versions of these models will provide "fundamental building blocks" for a large class of logarithmic CFTs.

The simplest of the (non-abelian) WZW models is that defined on $\mathsf{SU}(2)$. The symmetry algebra may be identified as the affine Kac-Moody algebra $\widehat{\mathfrak{sl}}(2)_k$, where the subscript k (the level) denotes the common eigenvalue of the generator of the (one-dimensional) centre. The geometric definition as a string theory restricts k to a non-negative integer [34]. However, one can try to construct the CFT algebraically for any real k if one gives up the geometric interpretation.

Such constructions are easiest for the so-called admissible levels. These are certain rational k for which the representation theory of $\widehat{\mathfrak{sl}}(2)_k$ behaves similarly to $k \in \mathbb{N}$ [35]. The corresponding CFTs were originally posited as non-unitary versions of WZW models from which the non-unitary minimal models could be constructed as cosets. However, further study of non-unitary WZW models found puzzling contradictions which led to a general feeling that these models suffered from an "intrinsic sickness" (see for example, [1, Sec. 18.6]).

This sickness was cured by Gaberdiel [36] who showed that the root cause was the assumption that non-unitary WZW models are rational CFTs. Indeed, for $k = -\frac{4}{3}$, he proved that the theory is only quasi-rational and moreover, logarithmic. This was followed up for $k = -\frac{1}{2}$ which admits a quasi-rational non-logarithmic construction [26, 37] and a logarithmic cover [38]. Aside from these two special values however, nothing further is known about the nature or structure of admissible-level WZW models.

This gap in our knowledge is clearly unacceptable if we accept the maxim that these models are the fundamental building blocks of logarithmic CFT. I propose to undertake a systematic investigation of such theories with the primary aim of filling this gap. To analyse specific models, the fusion algorithm of Nahm [31] and Gaberdiel-Kausch [4] will be programmed and applied to admissible representations in order to uncover the detailed logarithmic structure of these theories. I expect that the richer structure of $\widehat{\mathfrak{sl}}(2)$, as compared to the Virasoro algebra say, will lead to readily discernible patterns. Free-field methods will then be employed to prove these patterns. Other CFT phenomena (for example, modularity) will be generalised to these theories.

I also intend to investigate logarithmic versions of the $k \in \mathbb{N}$ theories. There is a little literature pertaining to the $k = 0$ case [39, 40] which makes it clear that there are potential applications for such theories. Investigation is anticipated to proceed as in the admissible cases, although here there are fewer constraints on the structure and inspiration will be sought from applications. It is also very interesting to investigate non-unitary WZW models with other symmetry algebras. However, such investigations are not envisaged to be as crucial as the $\widehat{\mathfrak{sl}}(2)$ case (see however sub-project 6).

4. Cosets of Logarithmic CFTs and Extended Algebras

An important (and indeed motivating) application of the development of non-unitary WZW models which I will follow is the construction of their coset theories. Recall that the admissible-level models were originally posited as a way to construct non-unitary minimal models via the coset mechanism. As it has since been realised that the admissible-level WZW models are generically logarithmic, the following question becomes pertinent: Do the (appropriate) cosets of the admissible-level WZW models recover the non-unitary minimal models, or some logarithmic version thereof? This simple question will be of significant interest to the logarithmic CFT community as much of the recent motivation in this area has come from attempts to construct logarithmic versions of minimal models. I aim to answer this question, which will hopefully shed important light on the structure of these logarithmic minimal models.

Logarithmic minimal models have their beginnings with the triplet model of Gaberdiel and Kausch [4–6]. This theory is formally associated with the empty (non-existent) minimal model $\mathcal{M}(1,2)$ and so is often denoted by $\mathcal{LM}(1,2)$. There are fairly straight-forward generalisations of the triplet model corresponding to $\mathcal{LM}(1,q)$, but the recent interest has come from trying to make sense of logarithmic minimal models $\mathcal{LM}(p,q)$ with $p > 1$ [14, 16, 19, 20] (unlike the $(1,q)$ models, these do have genuine minimal model analogues). Despite much effort, the status of these models remains somewhat questionable. Even their internal consistency is not clear (sub-project 5).

One thing which is generally agreed upon is that logarithmic minimal models should have an extended algebra (or W-algebra) with respect to which there are only finitely many representations. This is true for $\mathcal{LM}(1,q)$. Unfortunately, this extended algebra has not been explicitly determined in any other cases! Much has been deduced indirectly, or conjectured by analogy to the $(1,q)$ cases, but we are still missing the foundation upon which the study of logarithmic minimal models is based. I am currently writing up work in which I naturally reconstruct the W-algebra of the triplet model $\mathcal{LM}(1,2)$ using coset methods. I propose to generalise this result, using the results of sub-project 3, to the other logarithmic minimal models. This is expected to require detailed knowledge of both the admissible and integer-level logarithmic $\widehat{\mathfrak{sl}}(2)$ CFTs.

The representation theory of the resulting W-algebras will clearly play a fundamental role in analysis logarithmic minimal models. General W-algebra representations can be surprisingly rich in structure [41], so a careful study is warranted. This will involve proving mathematical structure theorems as well as computing fusion rules using the algorithm of Nahm and Gaberdiel-Kausch. I expect that by realising these W-algebras as cosets of simpler algebras, I will be able to exploit the known structure of the latter in determining the structure of the former. This provides additional motivation to follow this line of study.

5. Logarithmic Structure and the Adjoint

The chiral symmetry algebra of a CFT always comes equipped with an adjoint. This bestows upon the representations (the quantum state space) a hermitian form, invariant under the symmetry algebra action. This form is not necessarily positive-definite in general[3], but it plays the role of an inner product, describing the overlap between quantum states and hence the form of the physically relevant quantities, the correlation functions.

In a logarithmic CFT, it has always been assumed that the symmetry algebra comes equipped with the usual adjoint. However, we have some reason to be suspicious of this assumption. First, the zero-modes are generically self-adjoint in a CFT, so that the spectrum is real (hence physical). However, in a logarithmic CFT, some of these zero-modes cannot be diagonalised. This is not necessarily a contradiction as the hermitian form is indefinite on the state space, but one must now wonder why we need to choose an adjoint for which the zero-modes are self-adjoint in the first place. The choice of adjoint no longer seems fixed by physical considerations.

A more serious issue that this is related to is the following. As was noted in sub-project 4, there has been much recent activity in constructing logarithmic minimal models. However, it was realised that these constructions, though based on entirely reasonable physical models, lead to inconsistencies within the *standard* framework of CFT [11]. Specifically, these models lead to certain correlation functions which can be proven not to exist [42]. In essence, these correlation functions must satisfy three partial differential equations (PDEs) derived from the conformal Ward identities, and these PDEs have no common solution (not even 0). There are several possibilities to reconcile this inconsistency (see the discussion of [21] for some recent thoughts on this issue), but the relevant point here is that the derivation of the PDEs *assumes* the standard definition of adjoint.

I therefore propose to study the question of the adjoint in logarithmic CFT, with a view to resolve this current contradiction within the literature. I envisage the consideration of certain simple logarithmic theories and their physical applications to divine the physically relevant adjoint in such cases. If non-standard, this will be followed by reconstruction of the formalism of CFT so as to incorporate the modifications that such logarithmic adjoints require. It should be clear how crucial such a study is to physical applications of logarithmic CFT: Without a proper adjoint, we cannot have confidence that our computations of correlation functions have anything to do with the application at hand.

6. Supergroup WZW Models and D-Brane Charges

As mentioned above, integer-level WZW models describe strings propagating on a Lie group. Similarly, one can consider string theories defined on Lie supergroups [22–24], so-called supergroup WZW models. The corresponding CFT then has an affine Kac-Moody superalgebra for a symmetry algebra, representations of which almost always exhibit non-trivial indecomposable structure. It follows that these CFTs will be generically logarithmic. Supergroup WZW models and their cosets are currently objects of much interest, due to the role they play in the AdS/CFT correspondence [28]. Their logarithmic structure is however not usually remarked upon.

In contrast with the Lie group case, integer-level WZW models on Lie supergroups are almost never rational. From an algebraic point of view, the simplest case to analyse is actually when the level is admissible (see sub-project 3 for terminology). This case need not have a geometric interpretation as a string theory, but will provide further fundamental examples of logarithmic CFTs which exemplify

[3] What we therefore should say is that instead of an adjoint, we have an antilinear anti-involution with respect to which the induced hermitian form is contravariant.

the types of indecomposable representation which will be relevant more generally. I therefore propose that a first study concentrates on admissible-level supergroup WZW models.

Subsequent research will then focus on supergroup WZW models as string theories. Here, one possible application would be to extend to these models the well-known story concerning the classification of D-branes in standard WZW models by a certain twisted K-theory (described in [43, 44]). This is embodied in a celebrated theorem of Freed, Hopkins and Teleman [45] describing a natural isomorphism between this K-theory and the Verlinde (fusion) ring of the WZW model CFT. A generalisation of this is beyond the scope of the project, but the related issue of computing the classifying D-brane charges in supergroup WZW models should be amenable to analysis [46–48]. If successful, this could lead to a similar explosion of valuable activity at the interface of algebraic topology and string theory/CFT. It will be extremely interesting to see the role that logarithmic structure plays in this development.

The proposal which I have outlined above is substantial (as befits a five year fellowship), but one which promises to make significant contributions to cutting-edge research in logarithmic conformal field theory, cementing Australia's reputation for world-class research in this important discipline. As mentioned above, I expect that many more applications, to mathematics and physics, will become apparent during the course of this research. Undoubtedly, most will result from discussing my work with colleagues, both locally (in my department and nationally through conferences and workshops) and globally (via international conferences and collaborations).

A (very) tentative timetable outlining envisaged start/end dates for the sub-projects discussed here is as follows:

Year 1: Start projects 1, 2, 3 and 5.
Year 2: Continue projects 1, 2, 3 and 5. Start project 4.
Year 3: Continue projects 1, 2, 3 and 4. Start project 6
Year 4: Continue projects 1, 3, 4 and 6.
Year 5: Continue projects 4 and 6.

The space allocated has been deliberately reduced in later years to accommodate sub-projects not described here. In particular, projects centered on integrable perturbations of logarithmic CFTs and connections to SLE are candidates for such later work, and of course there are always new directions that become apparent during the course of any research.

It is of course necessary to acknowledge the possibility that various parts of this proposal will be found to be more difficult than anticipated. This proposal is based on preliminary results which have already uncovered surprising subtleties, so it would be folly to assume that this fellowship will see nothing but smooth sailing. However, I would hasten to add that the broad applicability of the methods being developed ensure that there will always be a multitude of different tasks to work profitably upon (and again I expect that more will present themselves as this research progresses). I am therefore confident that the programme I have outlined above will not suffer from the problem of getting stuck at a key point, as a narrower proposal perhaps might.

I submit that this represents an intensive, yet detailed, proposal of research during the period of the fellowship. It should be clear that this outlines an important unifying framework for logarithmic conformal field theory that deserves study, emphasises applicability, and that it will lead to many beautiful, interesting and useful mathematical results along the way.

E5: National Benefit

As mentioned above, the completion of the projects proposed here will lead to importance advances in the study and applications of logarithmic conformal field theory. I will develop precise algebraic analyses of fundamental examples of logarithmic conformal field theories and use this knowledge to construct a unifying framework in which to understand these models mathematically and discuss their applications physically.

The benefits of this programme of research are legion. Given the incredible amount of mathematical activity generated by the introduction of the Virasoro and Kac-Moody algebras some forty years ago, it is hard to believe that a study of their representations beyond the irreducible ones will not find similar applications in unifying areas of mathematics. The time is also ripe to exploit the structure of these representations in applications to condensed matter physics, string theory and SLE. Such results will be of great interest to the mathematical community, and will certainly help to foster further collaboration between physicists and mathematicians.

CFT is an active area of research, both in Australia and internationally. Logarithmic CFT is a rapidly advancing outgrowth of this discipline whose importance demands recognition. The outcomes described in this proposal will be of considerable interest to mathematical physicists and representation theorists across the country. I submit that funding this proposal will result in research of significant international standing. Moreover, the dissemination of the results of work proposed here will significantly enhance Australia's international reputation in the fundamental sciences.

E6: Communication of Results

As a mathematical physicist, my primary means of disseminating my work is through posting articles on the arXiv (`arxiv.org`), and then publishing them in the best reviewed journals. For example, my recent papers have been published in the Journal of High Energy Physics, Nuclear Physics B and Physics Letters B, each of which is among the very best in my field as evidenced by their 2007 impact factors: 5.659, 4.645 and 4.189 respectively. I expect that the work reported here will result in an average of at least four papers per year. I also take the opportunity to present my work at national and international workshops and conferences, and by giving seminars in other centres and departments. The nature of my field, consisting of abstract fundamental research, means that it is not appropriate to espouse direct communication with the public. However, there should exist opportunities to contribute to the training of students, locally at the departmental level, as well as nationally through schools and workshops such as those organised by AMSI and ICE-EM.

It is, I think, almost universally agreed amongst my peers that fundamental theoretical results of the nature we discover should be made available to the wider academic community. This is reflected in our primary means of dissemination, the internet, in that our ideas are never withheld (as long as one has an internet connection). Effectively, anybody may use my results, with the implicit understanding that my contribution will be acknowledged in their subsequent work.

E7: Role of Personnel

This is an application for an Australian Research Fellowship with a single applicant. As such, I must assume responsibility for all major aspects of the proposal described above. As an experienced researcher in mathematical physics, I claim to have (or be able to learn or develop) all the necessary expertise to satisfactorily complete this project. To support this claim, I refer to my track record as evidenced by my publications. Whilst this is discussed in detail elsewhere (section B10), I think it fair to say that my recent articles prove that I have considerable expertise in logarithmic conformal field theory and the relevant mathematical disciplines which will be required for this project. I would also like to think that they show that my research has been characterised by an ability to recognise difficult outstanding problems in mathematical physics, and solve them through the application of original mathematics. This should demonstrate that I certainly possess the capacity to undertake the research outlined above.

I also intend to actively involve postgraduate and honours students in various parts of this proposal. Choosing an appropriate topic will of course depend heavily on the student's interest and mathematical background. A preference for algebra and representation theory would be well accommodated in projects 1 and 4. Someone with a background in analysis might prefer to study the links with SLE. Geometric types could work on project 6. Furthermore, there is ample opportunity throughout for physical CFT research.

Aside from those funded by this proposal, there are many other experts, both in Australia and internationally, with whom I hope to collaborate. For example, I envisage continuing my present collaboration with Kalle Kytölä whilst pursuing project 1 and links to SLE, and I am sure that Pierre Mathieu will want to revisit the subject of extended algebras (project 4). Local expertise will also play a significant role in attaining these outcomes. For example, the experience of Paul Pearce will be a valuable resource in general (and for project 5 in particular), and that of Peter Bouwknegt in free field methods and W-algebras is particularly suited to projects 2 and 4.

Finally, there is a minor technical role to be played, essentially to provide support for the CPU-intensive algorithms which will need to be programmed. This role is discussed in more detail in C2.

E8: References

[1] P Di Francesco, P Mathieu, and D Sénéchal. *Conformal Field Theory*. Springer-Verlag, New York, 1997.
[2] L Rozansky and H Saleur. *Nucl. Phys.*, B376:461–509, 1992.
[3] V Gurarie. *Nucl. Phys.*, B410:535–549, 1993.
[4] M Gaberdiel and H Kausch. *Nucl. Phys.*, B477:293–318, 1996.
[5] M Gaberdiel and H Kausch. *Phys. Lett.*, B386:131–137, 1996.
[6] M Gaberdiel and H Kausch. *Nucl. Phys.*, B538:631–658, 1999.
[7] M Flohr. *Int. J. Mod. Phys.*, A18:4497–4592, 2003.
[8] R Langlands, P Pouliot, and Y Saint-Aubin. *Bull. Amer. Math. Soc.*, 30:1–61, 1994.
[9] J Cardy. *J. Phys.*, A25:L201–L206, 1992.
[10] R Langlands, C Pichet, P Pouliot, and Y Saint-Aubin. *J. Stat. Phys.*, 67:553–574, 1992.
[11] P Mathieu and D Ridout. *Phys. Lett.*, B657:120–129, 2007.
[12] J Fjelstad, J Fuchs, S Hwang, A Semikhatov, and I Yu Tipunin. *Nucl. Phys.*, B633:379–413, 2002.
[13] B Feigin, A Gainutdinov, A Semikhatov, and I Yu Tipunin. *Comm. Math. Phys.*, 065:47–93, 2006.
[14] B Feigin, A Gainutdinov, A Semikhatov, and I Yu Tipunin. *Nucl. Phys.*, B757:303–343, 2006.
[15] N Read and H Saleur. *Nucl. Phys.*, B613:409–444, 2001.
[16] P Pearce, J Rasmussen, and J-B Zuber. *J. Stat. Mech.*, 0611:017, 2006.
[17] N Read and H Saleur. *Nucl. Phys.*, B777:316–351, 2007.
[18] J Rasmussen and P Pearce. *J. Stat. Mech.*, 0709:002, 2007.
[19] H Eberle and M Flohr. *J. Phys.*, A39:15245–15286, 2006.
[20] P Mathieu and D Ridout. *Nucl. Phys.*, B801:268–295, 2008.
[21] D Ridout. *Nucl. Phys.*, B810:503–526, 2009.
[22] H Saleur and V Schomerus. *Nucl. Phys.*, B734:221–245, 2006.
[23] H Saleur and V Schomerus. *Nucl. Phys.*, B775:312–340, 2007.
[24] G Gotz, T Quella, and V Schomerus. *JHEP*, 0703:003, 2007.
[25] Y-Z Huang, J Lepowsky, and L Zhang. *Int. J. Math.*, 17:975–1012, 2006.
[26] D Ridout. `arXiv:0810.3532`.
[27] K Vogeler and M Flohr. `arXiv:0902.0729`.
[28] J Maldacena. *Adv. Theo. Math. Phys.*, 2:231–252, 1998.
[29] K Kytölä. `arXiv:0804.2612`.
[30] M Gaberdiel. *Int. J. Mod. Phys.*, A9:4619–4636, 1994.
[31] W Nahm. *Int. J. Mod. Phys.*, B8:3693–3702, 1994.
[32] F Rohsiepe. `arXiv:hep-th/9611160`.
[33] P Bouwknegt, J McCarthy, and K Pilch. *Prog. Theo. Phys. Supp.*, 102:67–135, 1990.
[34] E Witten. *Comm. Math. Phys.*, 92:455–472, 1984.
[35] V Kac and M Wakimoto. *Proc. Nat. Acad. Sci. USA*, 85:4956–4960, 1988.
[36] M Gaberdiel. *Nucl. Phys.*, B618:407–436, 2001.
[37] F Lesage, P Mathieu, J Rasmussen, and H Saleur. *Nucl. Phys.*, B647:363–403, 2002.
[38] F Lesage, P Mathieu, J Rasmussen, and H Saleur. *Nucl. Phys.*, B686:313–346, 2004.
[39] J Caux, I Kogan, A Lewis, and A Tsvelik. *Nucl. Phys.*, B489:469–484, 1997.
[40] A Nichols. *Phys. Lett.*, B516:439–445, 2001.
[41] P Bouwknegt, J McCarthy, and K Pilch. *The $\mathcal{W}_3$ Algebra: Modules, Semi-Infinite Cohomology and BV Algebras*. Springer, Berlin, 1996.
[42] V Gurarie and A Ludwig. *J. Phys.*, A35:L377–L384, 2002.
[43] E Witten. *JHEP*, 9812:019, 1998.
[44] P Bouwknegt and V Mathai. *JHEP*, 0003:007, 2000.
[45] D Freed, M Hopkins, and C Teleman. `arXiv:math.AT/0312155`.
[46] S Fredenhagen and V Schomerus. *JHEP*, 0104:007, 2001.
[47] P Bouwknegt, P Dawson, and D Ridout. *JHEP*, 0212:065, 2002.
[48] P Bouwknegt and D Ridout. *JHEP*, 0405:029, 2004.