Supplementary Information

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and Claus-Dieter Ohl[e]

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Variation of terminal velocity with bubble size

The measured terminal velocity, V_T of bubbles with diameters between 0.7 mm to 1.5 mm have been measured are shown in Fig. S1. This corresponds to Reynolds number, *Re* in the range 60 to 230.

Treating the bubbles as spheres of radius, R_o with the tangentially immobile boundary condition on the air-water interfaces, we can balance the buoyancy force:

$$F_{buoy} = \frac{4\pi}{3} \Delta\rho R_o^3 g \qquad\qquad (S1)$$

against the steady state hydrodynamic drag forces [1]:

$$F_{drag} = \frac{1}{2} \rho V_T^2 (\pi R_o^2) C_D \qquad\qquad (S2)$$

The drag coefficient C_D as a function of the Reynolds number: $Re = 2R_o \rho V_T/\mu$, where ρ is the density and μ is the shear viscosity of water.

We use the Schiller-Naumann formula [1] for C_D that is valid for $Re < 800$ with an error of less than $\pm 5\%$:

$$C_D = \frac{24}{Re}(1 + 0.15\,Re^{0.687}) \qquad\qquad\qquad\qquad (S3)$$

Combining eqs. (S1) - (S3) we can determine the terminal velocity as a function of bubble size. A comparison with our experimental values is given in Fig. S1. The excellent agreement indicates that the bubble-water interface is tangentially immobile.
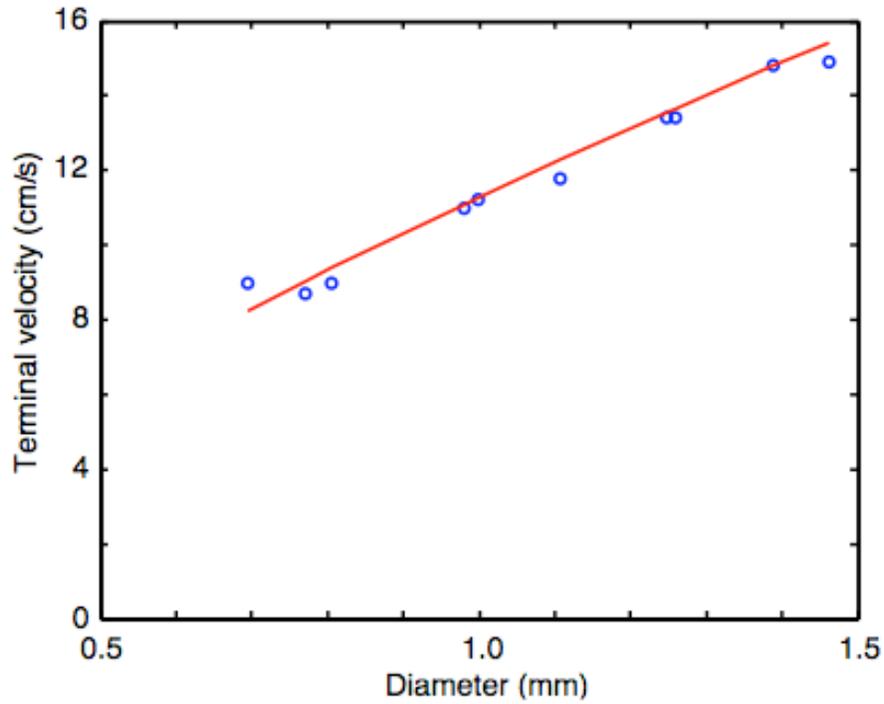

Fig. S1 Terminal velocity of spherical bubbles as a function of bubble diameter. Points: measured values. Line: Calculated values assuming the bubbles are 'solid' spheres obeying the tangentially immobile boundary condition.

Film thinning and hydrodynamic boundary condition

In Fig. S2, we compare the measured values of the water film thickness trapped between the bubble and the glass surface. Shown are the thickness at the centre of the film: $h(r=0, t)$ and the minimum thickness at the dimple rim: $h(r_{rim}, t)$.

Also shown are comparisons of the predicted film thickness according to lubrication theory using the tangentially immobile boundary condition and the zero tangential stress boundary condition at the bubble surface in which the film drainage equation

$$\frac{\partial h}{\partial t} = \frac{1}{3\mu} \frac{\partial}{\partial r}\left(r h^3 \frac{\partial p}{\partial r}\right) \qquad (S4)$$

This has a factor of 3 rather than 12 on the denominator of the RHS.

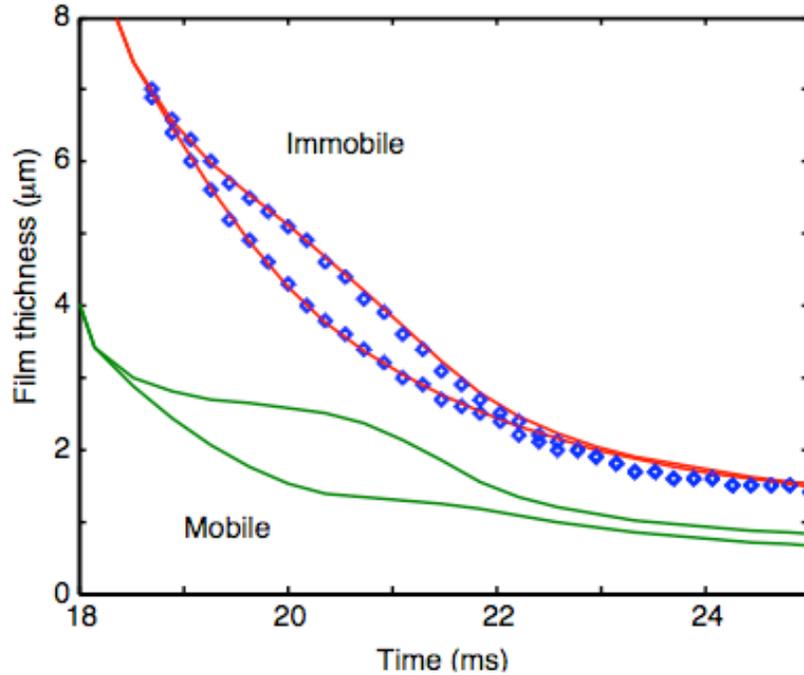**Fig. S2** Water film thickness at the centre: $h(r=0, t)$ and at the thinnest part, the dimple rim: $h(r_{rim}, t)$. Points: Experimental values determined from interference fringes. Lines: Predicted by lubrication theory using the tangentially immobile boundary condition at the bubble-water interface that gave excellent agreement with theory (see main text). Also shown are the smaller values of $h(r=0, t)$ and $h(r_{rim}, t)$ predicted by using the zero tangential stress condition at the bubble surface.

Extracting separation from fringes

The local film thickness $h(r,t)$ is obtained using the Bragg equation for a fringe of order m: $h = m (\lambda/2\,n)$, where $\lambda = 532$ nm is the wavelength of the laser and $n = 1.33$ is the refractive index of water. This means the difference in separation between 2 white fringes is 200 nm.

Building the shape profile in the top panel of Fig. S3 now becomes an exercise in identifying the center and the radial position of the rim and counting fringes. The film rim is the position where the film is thinnest and the separation would become larger on either side. The counting process is performed using Matlab to identify maxima and minima of the intensity. And by running this process over the entire movie we can extract the evolution of the profile with time. To get the absolute separation, we use the point of film rupture to determine the reference $h = 0$.
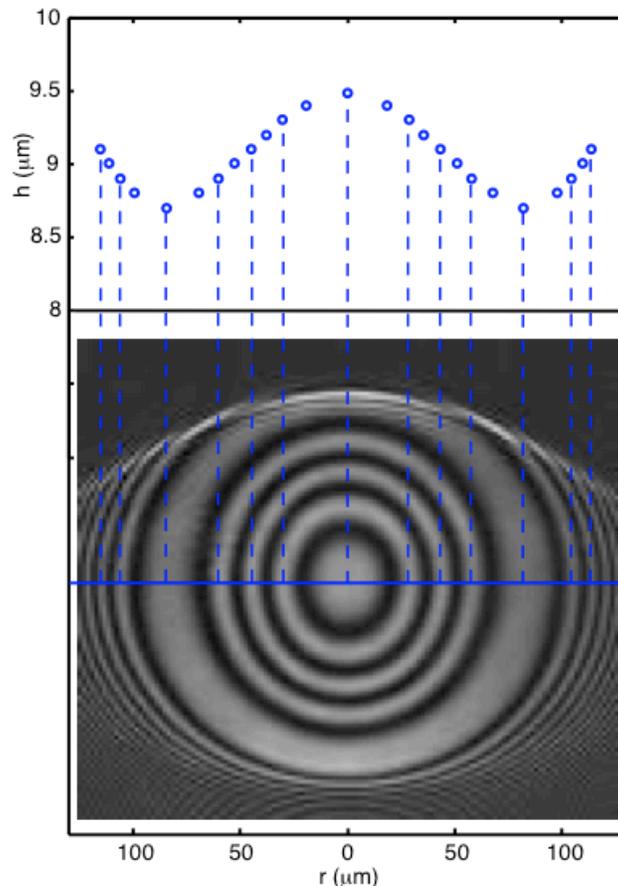

Fig. S3 Typical video frame containing intensity fringes and conversion to separation using the Bragg equation.

[1] R. Clift, J. R. Grace and M. E. Weber *Bubbles, Drops and Particles* Academic Press NY (1978).