Electronic Supplement – *Physical Review Letters*

Universal behavior of the initial stage of drop impact

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0 Introduction

Model drop falling under gravity onto a horizontal solid surface.

Detailed derivation of the governing equations

Scope

- Drop is an inviscid fluid under gravity moving towards solid surface
- Only consider motion of air in the thin gap between the drop and surface by Stokes flow Reynolds lubrication but omit motion elsewhere
- Drop has constant surface tension and deformation governed by the normal stress balance and normal pressure
- Consider the initial phase of the collision e.g. splashing not modeled.

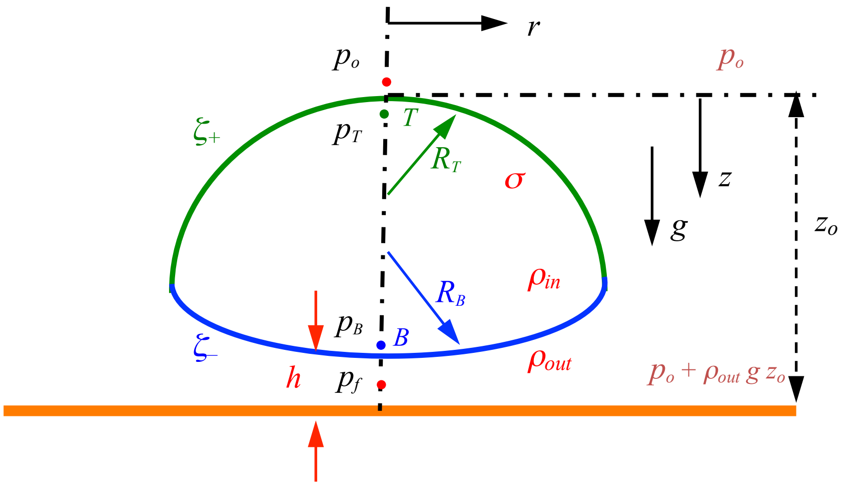

Figure 1. The shape of a drop impacting on a flat horizontal surface, where the theoretical parameters of the system are defined.

1 Normal stress balance - general

For a drop in equilibrium or an inviscid fluid, the normal stress is just the pressure.

$$P_{in} - P_{out} = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{1.1}$$

where σ is the interfacial tension. The radii of curvature R_1 and R_2 are defined to be positive when the concave side is the interior, when the pressure inside, P_{in}, is higher than the pressure outside, P_{out}.

2 Bernoulli equation

The momentum equation for an inviscid incompressible fluid in constant gravity is (see Fig 1)

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = -\nabla p + \rho g\,\nabla z \tag{2.1}$$

For potential flow, we have

$$\boldsymbol{u} = \nabla\phi \tag{2.2}$$

and incompressibility gives

$$\nabla^2\phi = 0 \tag{2.3}$$

So 2.1 becomes

$$\nabla\left(\rho\frac{D\phi}{Dt} - \tfrac{1}{2}\rho\, u^2\right) = \nabla(-p + \rho g z) \tag{2.4}$$

Integrating from some reference position $\boldsymbol{r}_o$ gives the Bernoulli equation

$$\begin{aligned}\left(\rho\frac{D\phi}{Dt} - \tfrac{1}{2}\rho\, u^2\right) - \left(\rho\frac{D\phi}{Dt} - \tfrac{1}{2}\rho\, u^2\right)_{r_o} \\ = (-p + \rho g z) - (-p + \rho g z)_{r_o} + p^*(t)\end{aligned} \tag{2.5}$$

where $p^*(t)$ is a position independent pressure that can vary with time.

For a falling drop in a quiescent medium $p^*(t) = 0$.

3 Stokes-Reynolds lubrication film

When the film Reynolds number $Re_f = \rho\ V\ (h\ R_o)^{1/2}/\mu << 1$ is small we can use the classical Reynolds lubrication equation to describe the evolution of the thickness $h(r,t)$. of the axisymmetric film of the external fluid trapped between the flat bottom of the drop and solid surface

$$\frac{\partial h}{\partial t} = \frac{m}{12\mu}\frac{1}{r}\frac{\partial}{\partial r}\left(r\,h^3\frac{\partial p_f}{\partial r}\right) \qquad 3.1$$

The hydrodynamic boundary condition on the solid is immobile. If the boundary condition on the drop surface is also immobile, m =1. If the zero tangential stress condition holds on the drop surface, m = 4.

Outside the film, the pressure p_f approaches the pressure in the outer bulk phase. For the case in which the outer phase is air with negligibly small viscosity compared to the drop, we an omit solving the Bernoulli equation in the outer phase and assume it is static so outside the film:

$$p_f \to p_o + \rho g\ z_o \qquad 3.2$$

Note that the density ρ and the viscosity μ in this section is that of the outer bulk phase.

Since the film thickness varies on a much smaller scale compared to the drop motion, 3.1 should be used to give the time-stepping in $h(r,t)$ that determines the evolution of the position of the bottom of the drop where the film pressure p_f is obtained from the normal stress balance for the film given in Sec 5.

4 Shape of an axisymmetric drop

The upper surface of the drop is given by (Fig 1)

$$z = \zeta_+(r,t) \tag{4.1}$$

and the lower surface of the drop is given by

$$z = \zeta_-(r,t) \tag{4.2}$$

The curvature on the upper/lower surface is given by

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \pm\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\,(\partial\zeta_\pm/\partial r)}{[1+(\partial\zeta_\pm/\partial r)^2]^{1/2}}\right) \tag{4.3}$$

At the bottom of a drop around point *B* that is flat, we $(\partial\zeta_-/\partial r) \ll 1$, so we define the mean radius of curvature R_B at the bottom of the drop by

$$\frac{2}{R_B} \equiv \left(\frac{1}{R_1}+\frac{1}{R_2}\right)_B = -\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\,(\partial\zeta_-/\partial r)}{[1+(\partial\zeta_-/\partial r)^2]^{1/2}}\right) \approx -\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial\zeta_-}{\partial r}\right) \tag{4.4}$$

Since from geometry, the width of the gap, *h*, between the bottom of the drop and the horizontal surface is

$$h(r,t) = z_B - \zeta_-(r,t) \tag{4.5}$$

We can express the mean radius of curvature R_B in terms of *h*

$$\frac{2}{R_B} \equiv \left(\frac{1}{R_1}+\frac{1}{R_2}\right)_B \approx \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial h}{\partial r}\right) \tag{4.6}$$

If we are modeling 2 identical drop, the gap, *h*, between them is

$$h(r,t) = 2z_B - 2\,\zeta_-(r,t) \tag{4.7}$$

where z_B is now the distance form the top of one drop to the median plane between the drops and we have the result

$$\frac{2}{R_B} \equiv \left(\frac{1}{R_1}+\frac{1}{R_2}\right)_B \approx \frac{1}{2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial h}{\partial r}\right) \tag{4.8}$$

With a factor 1/2 on the RHS.

5 Normal stress balance for the film

Application of the normal stress balance at the bottom of the drop where the pressure inside the drop is p_B.

The pressure outside the drop in the film of thickness, h, comprise of a hydrodynamic pressure, p_f, and a disjoining pressure, $\Pi(h)$, that accounts for molecular scale interactions between the drop and the solid surface due, for example, to van der Waals attraction responsible for the drop wetting the solid surface.

Thus the Laplace eqn 1.1 takes the form

$$p_B - [p_f + \Pi(h)] = \frac{2\sigma}{R_B} \equiv \sigma \left(\frac{1}{R_1} + \frac{1}{R_2}\right)_B \qquad 5.1$$

where R_B is the mean radius of curvature of the bottom of the drop.

When the bottom of the drop is very flat, we have from 4.6 for a drop against a flat solid surface

$$\frac{2}{R_B} \approx \frac{1}{r}\frac{\partial}{\partial r}\left(r\, \frac{\partial h}{\partial r}\right) \qquad 5.2$$

So the normal stress balance for the film thickness is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\, \frac{\partial h}{\partial r}\right) = p_B - p_f - \Pi(h) \qquad 5.3$$

Eq 5.3 should be solved for to give the film pressure p_f that will be used in the Stokes-Reynolds equation 3.1.

The pressure p_B is to be obtained from the Bernoulli equation 2.5.

For the case of 2 identical drops, the normal stress balance for the film thickness is

$$\frac{1}{2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\, \frac{\partial h}{\partial r}\right) = p_B - p_f - \Pi(h) \qquad 5.4$$

6 Model for drop impact

- The drop is an inviscid fluid, so solve the Bernoulli equation inside the drop
- Air affects only in the thin gap between the drop and surface. Do not solve the Bernoulli equation for the bulk air phase. Assume it is static.
- The surface tension of the drop is constant

Bernoulli equation

Choose the reference position $\boldsymbol{r}_o$ to be at the top of the drop at position T (just inside the drop at $z = 0$) where the pressure is p_T that is related to the pressure, p_o, in the outer phase just above the apex of the drop (with mean radius of curvature, R_T) by

$$p_{in}(\boldsymbol{r}_o, t) \equiv p_T = p_o + \frac{2\sigma}{R_T} \qquad 6.1$$

so eq 2.5 becomes

$$p_{in}(\boldsymbol{r}, t) = p_o + \frac{2\sigma}{R_T} + \rho_{in} g z - \rho_{in}\left(\frac{1}{2}\, u^2 - \frac{D\phi}{Dt}\right) + \rho_{in}\left(\frac{1}{2}\, u^2 - \frac{D\phi}{Dt}\right)_T \qquad 6.2$$

with

$$\boldsymbol{u} = \nabla\phi \qquad 6.3a$$

$$\nabla^2\phi = 0 \qquad 6.3b$$

with boundary condition

$$p_{in} - p_{out} = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad 6.4a$$

and

$$p_{out} = p_o + \rho_{out}\, g\, z \qquad 6.4b$$

Combine 6.4a and 6.4b to give

$$p_{in} - p_o - \rho_{out}\, g\, z = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad 6.5$$

that holds on all points on the surface apart from along the bottom.

Solve 6.2 to 6.6 for the pressure in the drop at the bottom: p_B

If $\boldsymbol{X}$ is the position of a point on the boundary, it moves with surface velocity $\boldsymbol{u}(\boldsymbol{X})$

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{u}(\boldsymbol{X}) \qquad 6.6$$

Stokes-Reynolds equation

$$\frac{\partial h}{\partial t} = \frac{m}{12\,\mu_{out}} \frac{1}{r} \frac{\partial}{\partial r}\left(r\, h^3 \frac{\partial p_f}{\partial r}\right) \qquad 6.7$$

Use 6.7 to time step $h(r,t)$ forward in time with p_f from 6.9 below.

Normal stress balance - film

For a drop on a solid flat surface

$$h(r,t) = z_o - \zeta_{-}(r,t) \qquad 6.8$$

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial h}{\partial r}\right) = p_B - p_f - \Pi(h) \qquad 6.9$$

Use 6.9 to express p_f in terms of p_B, $\Pi(h)$ and spatial derivatives of $h(r,t)$ and used in 6.7 to step $h(r,t)$ forward in time.

Normal stress balance - drop

$$\pm \frac{\sigma}{r} \frac{\partial}{\partial r}\left(\frac{r\,(\partial \zeta_{\pm}/\partial r)}{[1 + (\partial \zeta_{\pm}/\partial r)^2]^{1/2}}\right) = p_{in} - p_o - \rho_{out}\, g\, z \qquad 6.10$$

This is the boundary condition at the surface of the drop needed to solve the Bernoulli equation in the interior of the drop.

The reference pressure p_o in the outer phase just above the apex of the drop is a constant and so can be set to zero.

7 Model in air

- Set the reference pressure in air above the drop apex: $p_o = 0$
- $\rho_{out} = \rho_{air} << \rho_{in}$, set: $\rho_{out}\, g\, z_o = 0$, so film pressure $p_f \rightarrow 0$ outside the film

The reference pressure p_o in the outer phase just above the apex of the drop is a constant and so can be set to zero.

Bernoulli equation

Eq 6.2 becomes

$$p_{in}(\boldsymbol{r},t) = \frac{2\sigma}{R_T} + \rho_{in} g z - \rho_{in}\left(\tfrac{1}{2}\, u^2 - \frac{D\phi}{Dt}\right) + \rho_{in}\left(\tfrac{1}{2}\, u^2 - \frac{D\phi}{Dt}\right)_T \qquad 7.1$$

with

$$\boldsymbol{u} = \nabla\phi \qquad 7.2a$$

$$\nabla^2\phi = 0 \qquad 7.2b$$

with boundary condition

$$p_{in}(\zeta) = \pm\frac{\sigma}{r}\frac{\partial}{\partial r}\left(\frac{r\,(\partial\zeta_{\pm}/\partial r)}{[1+(\partial\zeta_{\pm}/\partial r)^2]^{1/2}}\right) \qquad 7.3$$

that holds on all points on the surface.

Solve 7.1 to 7.3 for the pressure in the drop at the bottom: p_B

If $\boldsymbol{X}$ is the position of a point on the boundary, it moves with surface velocity $\boldsymbol{u}(\boldsymbol{X})$

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{u}(\boldsymbol{X}) \qquad 7.4$$

Stokes-Reynolds equation

$$\frac{\partial h}{\partial t} = \frac{m}{12\,\mu_{air}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\, h^3\frac{\partial p_f}{\partial r}\right) \qquad 7.5$$

$$h(r,t) = z_o - \zeta_{-}(r,t) \qquad 7.6$$

Normal stress balance - film

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial h}{\partial r}\right) = p_B - p_f - \Pi(h) \qquad 7.7$$

for a drop on a solid flat surface.

Use 6.9 to express p_f in terms of p_B, $\Pi(h)$ and spatial derivatives of $h(r,t)$ and used in 6.7 to step $h(r,t)$ forward in time.

Approximate p_B by the stagnation pressure

The point on the axis of symmetry at position $(r, z) = (0, z_B)$, at the bottom of the drop, is a stagnation point where the velocity is identically zero. We estimate the pressure at this stagnation point as follows. Assume the velocity at the top of the drop at position T is a constant V, then we have

$$p_{stag} = \ \tfrac{1}{2}\,\rho_{in} V^2 \qquad 7.8$$

We now approximate the pressure, p_B, along the bottom of drop with the inclusion of the stagnation pressure, p_{stag}, to give

$$p_B \ \cong \frac{2\sigma}{R_T} + \ \rho_{in} g z_B \ + \tfrac{1}{2}\,\rho_{in} V^2 \qquad 7.9$$

This is eq. (1) in the main text.

Characteristic scales

From 7.1

$$P \sim \frac{\sigma}{R} + \rho_{in}\, g\, R_o + \rho_{in}\, V^2 \qquad 7.10$$

From 7.5

$$V \sim \frac{1}{\mu_{air}} \frac{1}{R_o h} (h^3\, P) \qquad 7.11$$

Use 7.8 in 7.9

$$V \sim \frac{R_o}{\mu_{air}} \frac{h^2}{{R_o}^2} \left(\frac{\sigma}{R_o} + \rho_{in}\, g\, R_o + \rho_{in}\, V^2 \right) \qquad 7.12$$

Rearrange to give

$$\frac{h}{R_o} \sim \left(\frac{1}{Ca} + \frac{Eo}{Ca} + St \right)^{-1/2} \qquad 7.13$$

Capillary number, *Ca*

$$Ca = \frac{\mu_{air}\, V}{\sigma} \qquad 7.14$$

Eötvös number, *Eo* (or Bond number). Note that ρ_{out} is negligible for drop in air

$$Eo = \frac{\rho_{in}\, g\, {R_o}^2}{\sigma} \qquad 7.15$$

Stokes number, *St*

$$St = \frac{\rho_{in}\, R_o\, V}{\mu_{air}} \qquad 7.16$$

Weber number, *We*

$$Ca\, St = We = \frac{\rho_{in}\, R_o\, V^2}{\sigma} \qquad 7.17$$

Note *We* involves the density of the drop (internal phase).

8 Non-dimensional model

Define dimensionless number K

$$K \sim \left(\frac{1}{Ca} + \frac{Eo}{Ca} + St\right)^{-1} \equiv \frac{Ca}{1 + Eo + Ca\,St} \equiv \frac{Ca}{1 + Eo + We} \tag{8.1}$$

Scaled non-dimensional $O(1)$ variables denoted by over bar for a drop on a solid flat surface.

$$h \sim \left(2^{1/2}\, K^{1/2}\, R_o\right)\ \bar{h}$$

$$r \sim \left(2^{3/4} K^{1/4}\, R_o\right)\ \bar{r}$$

$$t \sim \left(2^{1/2} K^{1/2}\, R_o/V\right)\ \bar{t}$$

$$u \sim (V)\ \bar{u} \tag{8.2}$$

$$p_{in} \sim \left(\frac{\sigma}{R_o} + \rho_{in}\, g\, R + \rho_{in}\, V^2\right) \bar{p}_{in} \equiv P\, \bar{p}_{in}$$

$$F \sim K\ \left(\frac{\sigma}{R} + \rho_{in}\, g\, R + \rho_{in}\, V^2\right) R_o^2\ \bar{F} \equiv K\ P R_o^2\ \bar{F}$$

$$P \equiv \left(\frac{\sigma}{R_o} + \rho_{in}\, g\, R + \rho_{in}\, V^2\right)$$

Scaled Stokes-Reynolds equation

$$\frac{\partial \bar{h}}{\partial \bar{t}} = \frac{m}{12} \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r}\, (\bar{h})^3 \frac{\partial \bar{p}_f}{\partial \bar{r}}\right) \tag{8.3}$$

Scaled normal stress balance - film

$$\frac{1}{2} \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r}\ \frac{\partial \bar{h}}{\partial \bar{r}}\right) = \bar{p}_B - \bar{p}_f - \frac{1}{P} \Pi(h) \tag{8.4}$$

Scaled Bernoulli equation

$$\begin{aligned}&(1 + Eo + We)\, \bar{p}_{in} \\ &= 2\left(\frac{R_o}{R_T}\right) + Eo\ \left(\frac{z}{R_o}\right) + We\left(\frac{1}{2}\ \bar{u}^2 - \frac{D\bar{\phi}}{D\bar{t}}\right) - We\left(\frac{1}{2}\ \bar{u}^2 - \frac{D\bar{\phi}}{D\bar{t}}\right)_T\end{aligned} \tag{8.5}$$

For the case of the drop-drops, we use the scaling

$$
\begin{aligned}
h &\sim \left(K^{1/2}\, R_o \right)\, \bar{h} \\
r &\sim \left(K^{1/4}\, R_o \right)\, \bar{r} \\
t &\sim \left(K^{1/2}\, R_o / V \right)\, \bar{t} \\
u &\sim (V)\, \bar{u} \\
p_{in} &\sim \left(\frac{\sigma}{R_o} + \rho_{in}\, g\, R + \rho_{in}\, V^2 \right) \bar{p}_{in} \equiv P\, \bar{p}_{in} \\
F &\sim K \left(\frac{\sigma}{R} + \rho_{in}\, g\, R + \rho_{in}\, V^2 \right) R_o^2\, \bar{F} \equiv K\, P R_o^2\, \bar{F} \\
P &\equiv \left(\frac{\sigma}{R_o} + \rho_{in}\, g\, R + \rho_{in}\, V^2 \right)
\end{aligned}
\qquad 8.6
$$

In the respective scaled variables, the drop-solid and the drop-drop governing equations appear exactly the same.

9 Scaling of h_d

The scaling of h_d in eq. (10) of the main text is

$$h_d = 0.4\,R_o(2K)^{1/2} = 0.4\sqrt{2}\,R\,\frac{Ca^{1/2}}{(1+Eo+We)^{1/2}} \qquad 9.1$$

where

$$Ca = \frac{\mu_{air}\,V}{\sigma},\;\; Eo = \frac{\rho_{in}\,g\,{R_o}^2}{\sigma},\;\; We = \frac{\rho_{in}\,R_o\,V^2}{\sigma} \qquad 9.2$$

so that

$$\frac{dCa^{1/2}}{dV} = \frac{Ca^{1/2}}{2V},\qquad \frac{dWe}{dV} = \frac{2We}{V} \qquad 9.3$$

The maximum of h_d is located by

$$\frac{dh_d}{dV} = 0.4\sqrt{2}\,R_o\left\{\frac{Ca^{1/2}}{(1+Eo+We)^{1/2}}\frac{1}{2V} - \frac{Ca^{1/2}}{(1+Eo+We)^{3/2}}\frac{We}{V}\right\} = 0 \qquad 9.4$$

to give the velocity at the maximum V_{max}

$$\frac{\rho_{in}\,R_o\,{V_{max}}^2}{\sigma} = We_{max} = 1+Eo;\qquad V_{max} = \sqrt{\frac{\sigma(1+Eo)}{\rho_{in}\,R_o}} \qquad 9.5$$

Therefore the maximum dimple height $(h_d)_{max}$ is

$$(h_d)_{max} = 0.4\,R_o\,\frac{(Ca^{1/2})_{max}}{(1+Eo)^{1/2}} = 0.4\,R_o\,\frac{\mu^{1/2}}{[\rho_{in}\,\sigma R_o(1+Eo)]^{1/4}} \qquad 9.6$$

10 Experimental data of drop and bubble impact

In the following tables we summarize the experimental parameters from different sources used in for Figure 2 in the main manuscript for h_d the separation at which the dimple first develops.

Drop in air on glass

In Table S1, experimental parameters are from the experiment of Van der Veen et al [1] using water drops falling on a glass slide and also experiments of Bouwhuis et al [2] in which a wide range of impact velocities were studied for a ethanol drops falling onto glass.

Ref	Experiment	ρ_{out} (kg/m^3)	μ_{out} (μPa s)	ρ_{in} (kg/m^3)	μ_{in} (mPa s)	R (mm)	σ (mN/m)	V (m/s)	h_d (μm)
[1]	Drop-solid	1.22	18	1000	1.0	1.0	72	0.22	4
[2]	Drop-solid	1.22	18	780	1.0	0.9	22	10^{-3} - 2	0.2-5

Table S1. Liquid drops impacting on glass surface.

Mercury drop in water on mica

In Table S2, experimental parameters for experiments of Connor and Horn [3] on the interaction between a mercury drop and a mica surface in water.

Ref	Experiment	ρ_{out} (kg/m^3)	μ_{out} (mPa s)	ρ_{in} (kg/m^3)	μ_{in} (mPa s)	R (mm)	σ (mN/m)	V (μm/s)	h_d (μm)
[3]	Drop-solid	1000	0.9	13500	1.5	1.9	420	24	0.25
[3]	Drop-solid	1000	0.9	13500	1.5	1.9	420	67	0.4

Table S2. Liquid drops impacting surfaces or other drops in another liquid.

Bubble in water on glass

In Table S3, experimental parameters are for the impact of buoyant bubbles rising in water and hitting a glass surface. One experiment was performed using regular deionized water in which the boundary condition at the bubble surface during impact was immobile [4] [5]. The second experiment used cleaner water [6] and the boundary condition was mobile and that is the reason the dimple formation happening at separations that are shorter for similar sized bubbles.

Ref	Experiment	ρ_{out} (kg/m^3)	μ_{out} (mPa s)	ρ_{in} (kg/m^3)	μ_{in} (μPa s)	R (μm)	σ (mN/m)	V (cm/s)	h_d (μm)
[4]	Bubble-solid	1000	1.0	1.22	18	385	72	8.7	7
[5]	Bubble-solid	1000	1.0	1.22	18	630	72	13.4	18
[6]	Bubble-solid	1000	1.0	1.22	18	400	72	9.2	3.5
[6]	Bubble-solid	1000	1.0	1.22	18	625	72	13.5	7

Table S3. Air bubbles impacting a glass surface in water.

Drop-drop in another liquid

In Table S4, experimental parameters from Klaseboer et al [7] obtained using a combination of drop-drop impact in which water, glycerol and silicone oil drops were held by two syringes and driven against each other at low speed.

Ref	Experiment	ρ_{out} (kg/m^3)	μ_{out} (Pa s)	ρ_{in} (kg/m^3)	μ_{in} (Pa s)	R (mm)	σ (mN/m)	V (μm/s)	h_{d} (μm)
[7]	Drop-drop	1000	0.3	1000	1.0	1.5	30	2.0	2.7
[7]	Drop-drop	1000	0.2	1000	0.1	1.5	30	1.67	2.1
[7]	Drop-drop	1000	0.05	1000	1.0	1.5	30	6.7	2.0
[7]	Drop-drop	1000	0.1	1000	2.0	1.5	30	3.33	2.0
[7]	Drop-drop	1000	0.023	1000	0.05	1.5	30	2.67	0.85
[8]	Drop-drop	1000	0.3	1000	1.0	1.5	30	6.7	5.0

Table A4. Drop-drop interaction inside another liquid at low speeds.

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