EPAPS

Order from Chaos:
Dynamics of density segregation in continuously aerated granular systems

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Contents

1 Materials

Spherical glass beads (GB) of bed density ρ_{GB}= 1.5 gm/cm^3 and spherical iron powder (IP) of bed density ρ_{IP} = 4.5 gm/cm^3 were used as the powder bed media. The bed density ratio, ρ_{IP}/ρ_{GB} = 3.0. The diameters, d of the GB and IP were the same: 0.18−0.21 mm.

For experiments reported in the main text, the GB and IP were mixed at a 50:50 bulk volume fraction, that is V_{IP} = 50 % = V_{GB}, giving a bed density of the 50:50 GB+IP mixture of (1.5 + 4.5)/2 = 3.0 gm/cm^3.

Glass beads and iron powder mixtures of volume fraction V_{IP} = 30 % (i.e V_{GB} = 70 %) and V_{IP} = 70 % (i.e V_{GB} = 30 %) were also prepared to investigate the relationship between pressure drop and superficial air velocity.

The GB and IP belong to Geldart Class B non-cohesive, granular media [1].

2 Experimental apparatus

The experimental setup is shown in Fig S1 consists of a cylindrical acrylic column of inner diameter 100 mm and height 500 mm, an air distributor, and an air chamber. The air distributor was fabricated by clamping a piece of cloth between two perforated metal plates of thickness 1 mm that have an equilateral triangular mesh of holes of diameter 8 mm and pitch 10 mm. Compressed air was fed in the air chamber. The air was injected into the bottom of the container through the air distributor. A mass flow controller is used to set the superficial air velocity,

$$u = \text{(air volume per unit time) / (container cross-sectional area) [cm/s]}.$$

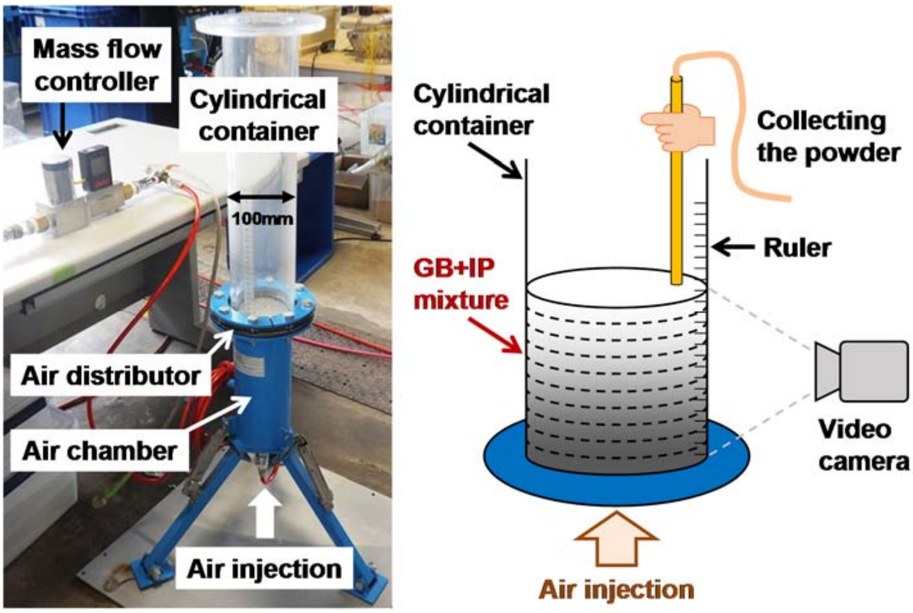


Figure S1 Photo and schematic of the experimental setup used for the density segregation experiments.

3 Determination of the minimum fluidization velocity

The minimum fluidization velocity, u_{mf}, that is the minimum superficial air velocity at which the powder bed begins to be fluidized, was measured for the GB, IP and the GB+IP mixtures as follows. The cylindrical container of diameter 100 mm was filled to a bed height of 100 mm with the GB, the IP, or the GB+IP mixtures, and air was injected into the fixed powder bed. The pressure drop, P across the powder bed was measured using a manometer as u is gradually increased.

3.1 *Fixed bed regime*: $0 < u < u_{mf}$

The pressure drop P increases linearly with u at from $u = 0$ to $u = u_{mf}$ that is characteristic of the flow of a Newtonian fluid flow through a rigid porous medium – the powder bed said to be the fixed bed region. The measured pressure-velocity relation obeys the Darcy equation for a fixed powder bed of height, h

$$P = (\mu h/K)\, u$$

where μ is the viscosity of the fluid (air = 1.98×10^{-5} Pa s). The permeability of the fixed bed, K is predicted by the Carmen-Kozeny equation [2]

$$K = 180\,(1 - \varepsilon)^2/(d^2\,\varepsilon^3)$$

with a porosity, $\varepsilon = 0.4$ that corresponds to that of a randomly pack bed of hard spheres of diameter, d.

3.2 *Fluidized bed regime*: $u > u_{mf}$

However, for $u > u_{mf}$, the pressure drop P remained constant that is characteristic of the constant pressure drop exhibited by low Reynolds number flow of a Newtonian fluid along a pipe. The powder bed is then said to be in a fluidized bed state in this velocity regime.

The minimum fluidization velocity u_{mf} is determined as the "transition point" between the linearly increasing fixed bed and the horizontal fluidized bed regimes of the P vs u data. Results for pure GB or pure IP or GB+IP mixtures at three different values of the volume fraction of IP, V_{IP} are shown in Fig 1 of the main text. The size of the data points represents the variation between three repeated measurements.

Values of the minimum fluidization velocity u_{mf} determined by extrapolation of the fixed bed and fluidized bed regime results are corresponding to the different powder systems in Fig 1 of the main text are shown in Table S1.

Table S1. Minimum fluidization velocity, u_{mf} of GB, IP and GB+IP mixtures

Powder bed	Pure IP	GB+IP mixture $V_{IP} = 70\%$	GB+IP mixture $V_{IP} = 50\%$	GB+IP mixture $V_{IP} = 30\%$	Pure GB
u_{mf} [cm/s]	u_{mf} (IP) ≡ 9.2	7.8	6.0	4.8	u_{mf} (GB) ≡ 3.1

The variation of the minimum fluidization velocity, u_{mf} with the volume fraction of the IP, V_{IP} and GB, $V_{GB} \equiv (1 - V_{IP})$ shown in Table S1 is well represented by the effective medium result

$$u_{mf}^{eff} \equiv u_{mf}\,(\mathrm{IP})\, V_{IP} + u_{mf}\,(\mathrm{GB})\, V_{GB}$$

3.3 *Scaled P vs u data*

The results for all powder systems scaled to a common master curve shown in the inset of Fig 1 of the main text if the velocity is scaled by u_{mf}^{eff} and the pressure is scaled by

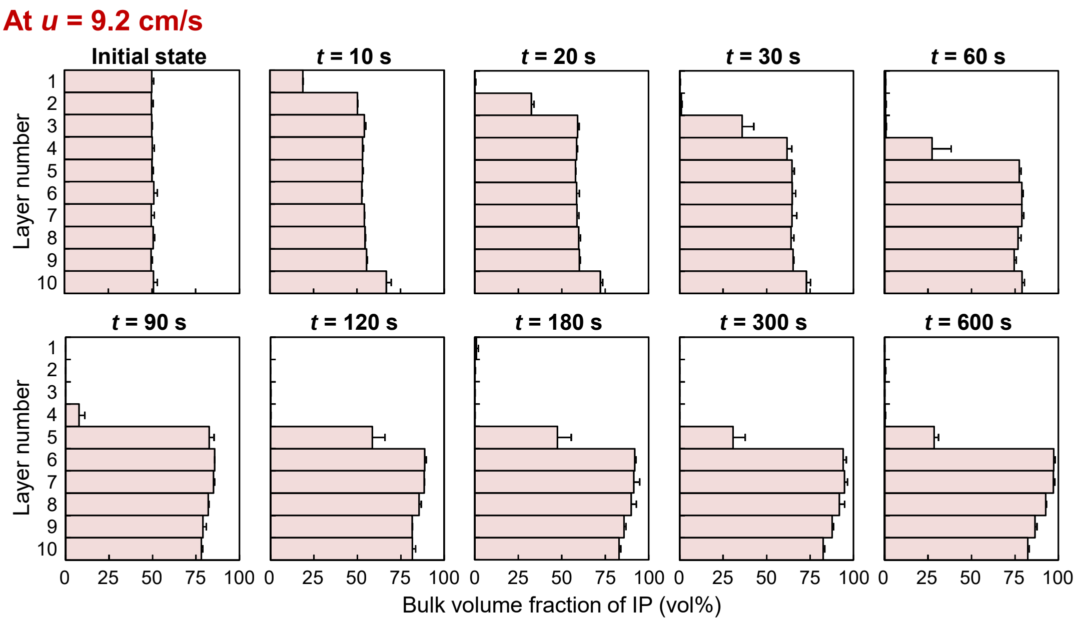
$$P_{mf} \equiv \rho_{bed}\, gh = (\rho_{IP}\, V_{IP} + \rho_{GB}\, V_{GB})\, gh$$

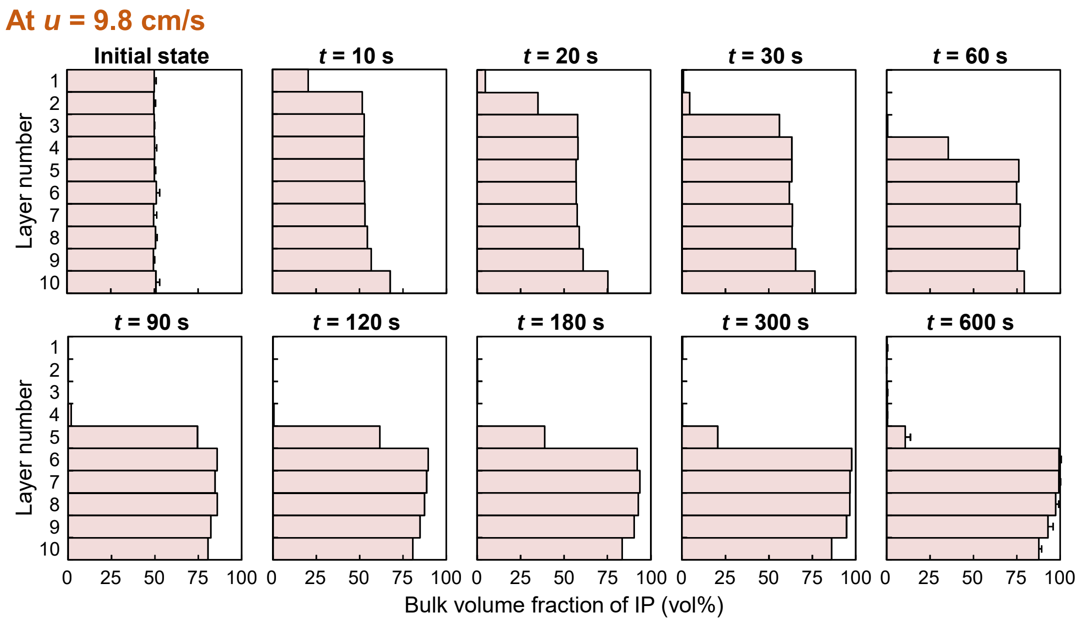
where g is the gravitational acceleration. The powder beds make the transition from a fixed bed to a fluidized bed when the pressure drop P is equal to the weight of the bed divided by the cross-sectional area.

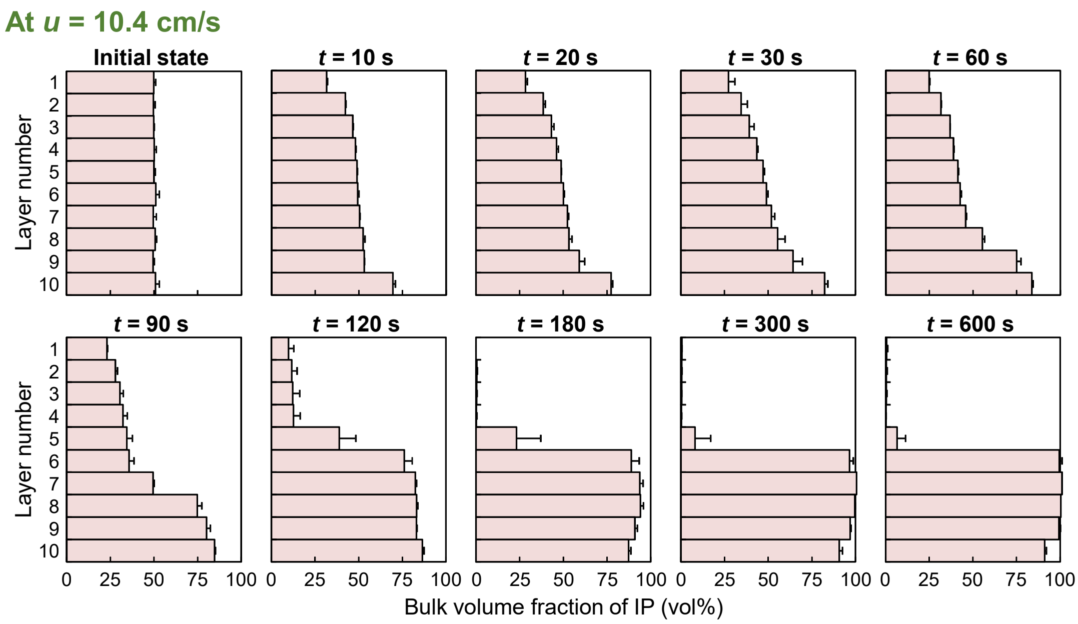
4 Density segregation experiment

The temporal and spatial variations of the dynamic of segregation were investigated using the experimental apparatus and schematic drawing shown in Fig S1 for a 50:50 GB+IP mixture (V_{IP} = 50 %). Initially, the cylindrical container was filled to a depth of 100 mm with the well-mixed GB+IP powder. Time and height variations of the density segregation dynamics at u = 9.2, 9.8, 10.4, 11.2, and 11.8 cm/s were recorded after the mixture has been fluidized for fixed time intervals between 10 s and 600 s.

After a fixed fluidization time, the aeration is stopped and successive layers of the powder bed of 10 mm thick were then collected by a vacuum cleaner. The powder bed density ρ at each layer was recorded from which the volume fraction of IP, V_{IP} at each layer can be calculated. This gives 10 layers of IP density *vs* height data for the 100 mm column at the given fluidization time. Some measurements were repeated three times to check for experimental variability. Repeating such measurements for different fluidization times then gives quantitative data for the temporal and spatial variations of the segregation process.


At u = 9.2 cm/s
Initial state
t = 10 s
t = 20 s
t = 30 s
t = 60 s
t = 90 s
t = 120 s
t = 180 s
t = 300 s
t = 600 s
Layer number
Bulk volume fraction of IP (vol%)


At u = 9.8 cm/s
Initial state
t = 10 s
t = 20 s
t = 30 s
t = 60 s
t = 90 s
t = 120 s
t = 180 s
t = 300 s
t = 600 s
Layer number
Bulk volume fraction of IP (vol%)


At u = 10.4 cm/s
Initial state
t = 10 s
t = 20 s
t = 30 s
t = 60 s
t = 90 s
t = 120 s
t = 180 s
t = 300 s
t = 600 s
Layer number
Bulk volume fraction of IP (vol%)

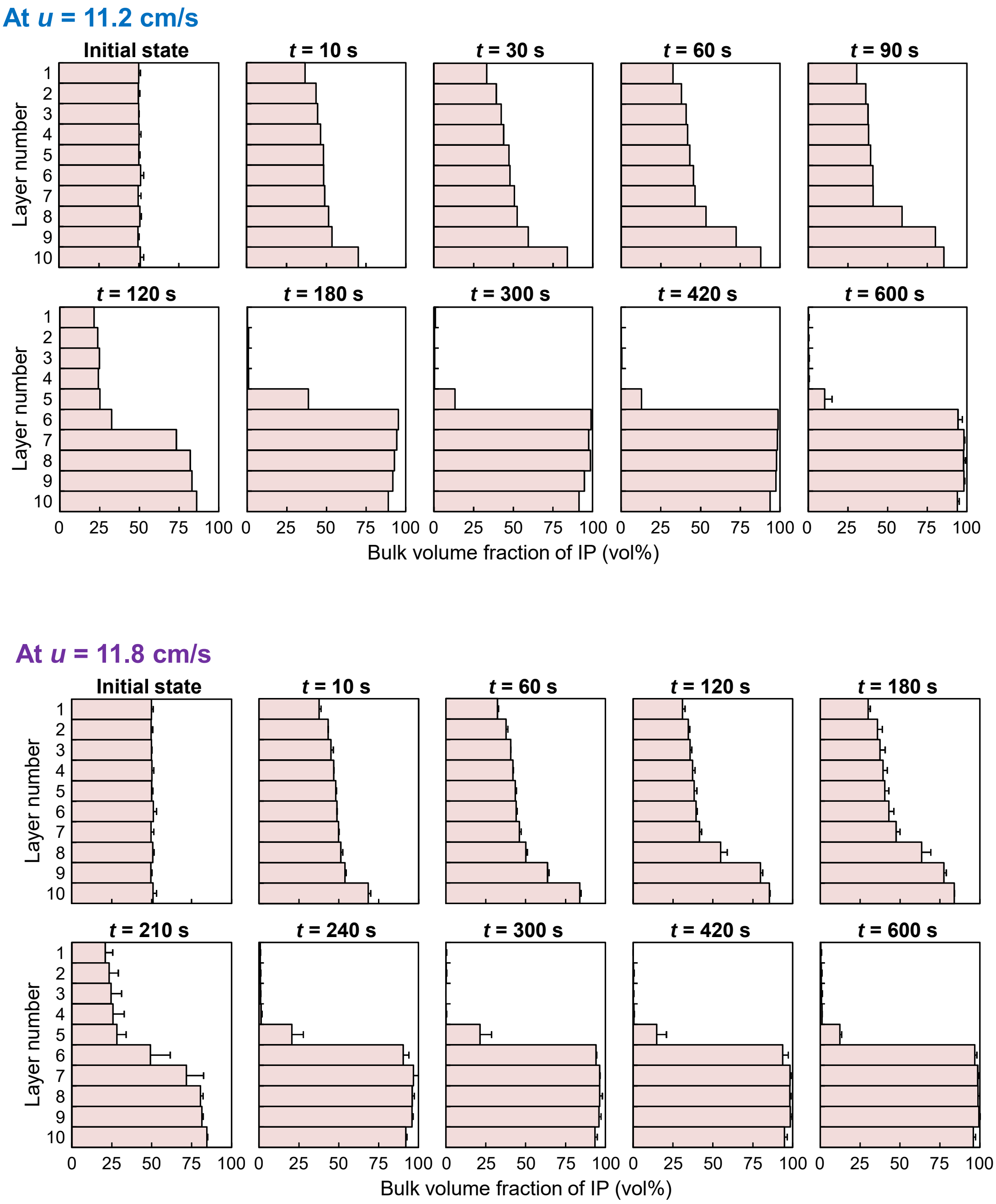


Figure S2 Experimental results of the variation of the iron powder volume fraction with depth in the powder bed starting from an initially uniform mixture of 50:50 glass beads (GB) + iron powder (IP) at air velocities u = 9.2, 9.8, 10.4, 11.2, and 11.8 cm/s.

Results for the variation of IP volume fraction at different layers down the bed with time for velocities u = 9.2, 9.8, 10.4, 11.2, and 11.8 cm/s are summarized in Fig S2. It is evident from these results that at all 5 air velocities, the bed segregates nearly completely after 600 s with the lighter GB migrated to the top half of the bed and the heavier IP to the bottom half.

However, there are 2 distinct modes of separation. At lower air velocities, u = 9.2 and 9.8, segregation is nearly complete after about 100 s, with the negligible IP present in the top 4 layers. At higher air velocities, u = 10.4, 11.2, and 11.8 cm/s, it takes around 240 s before the top 4 layers become free of the heavier IP.

4.1 *Initial iron powder flux*

The much shorter time needed for the heavier IP to migrate from the top 4 layers of the bed can be explained in terms of the initial flux of IP downwards and the corresponding flux of lighter GB to migrate upwards. The initial flux can be estimated from the difference between the originally uniform IP distribution of the 50:50 mixture and the IP distribution after 10 s of fluidization. The change in the IP volume fraction in layer L after 10 s, $\Delta V_{IP}(L)$ can be expressed in terms of the flux of IP, $J(L{-}1, L)$ from layer $(L{-}1)$ into layer L, where L is layer number: $L = 1$ denotes the top layer and $L = 10$ denotes the bottom layer:

$$\Delta V_{IP}(L) = J(L-1, L) - J(L, L+1).$$

Thus with the obvious conditions: $J(0, 1) = 0 = J(10, 11)$, the fluxes $J(L{-}1, L)$ can be calculated from the measured values of $\Delta V_{IP}(L)$. These resulting fluxes are given in the inset of Fig 4 in the main text.

The flux of GB can be obtained in the similar way since $V_{GB} + V_{IP} = 1$.

4.2 *Segregation rate and segregation efficiency*

Data for the variation of the IP volume fraction in the 10 layers in the powder bed after a fluidization time of 600 s at air velocities between 6 cm/s (the u_{mf} of the 50:50 GB+IP mixture) and 15 cm/s are given in Fig S3. As expected, segregation only occurs at air velocities is above the fixed bed regime. Segregation becomes most noticeable for air velocities above the minimum fluidization velocity of the pure IP, u_{mf}(IP) = 9.2 cm/s up to 12 cm/s, at which the upper 4 layers are completely devoid of IP and the volume fraction of IP in the lower 5 layers is nearly 100 %. However, we can see from the results in Fig S2 that within the velocity range 9.2 cm/s to 12 cm/s, it takes longer to achieve the same degree of segregation at higher air velocities. At even higher air velocities above 12 cm/s, no segregation occurs as the bed remained well mixed as expected.

For a given spatial variation of the layer composition of the 50:50 mixture, we can define the segregation efficiency, E of the entire column in terms of the percent volume fraction of iron powder in layer L, $V_{IP}(L)$ by

$$E = \frac{1}{10}\left\{\sum_{L=1}^{5}\frac{50 - V_{IP}(L)}{50} + \sum_{L=6}^{10}\frac{V_{IP}(L) - 50}{50}\right\} \times 100\ \%$$

For a well-mixed non-segregated mixture, $V_{IP}(L)$ = 50% for all layers, L and so E = 0. When the GB+IP mixture segregates perfectly into an upper half GB layers and a lower half IP layers, $V_{IP}(L)$ = 0% for the upper five layers, L = 1 to 5, and $V_{IP}(L)$ = 100% for the lower five layers, L = 6 to 10 then corresponds to E = 100%. The variation of E with air velocity, u after a fluidization time of 600 s is shown in Fig 3 of the main text.

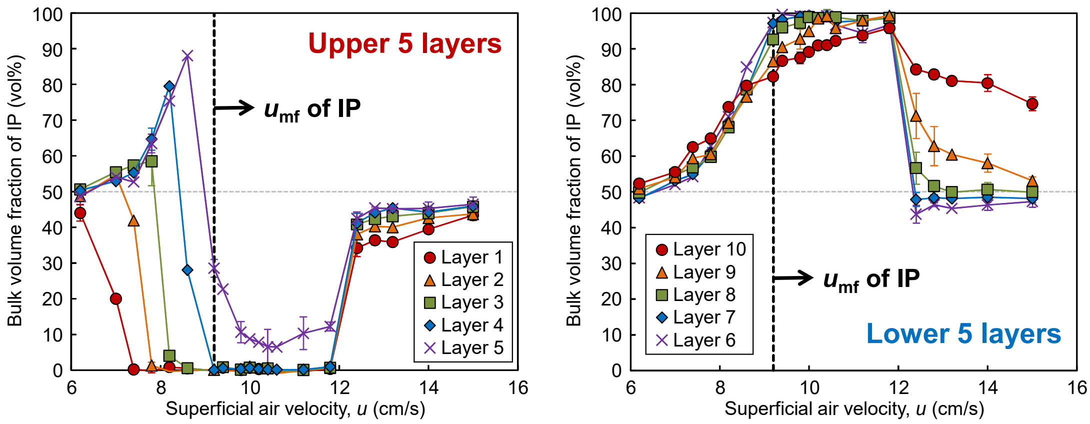


Figure S3 Variation of the iron powder (IP) volume fraction at layers L = 1 (top) to 10 (bottom) in the bed with air velocity, u after fluidizing for 600 s.

The measured variations in layer density with depth after various aeration times given in Fig S2 can now be processed to give the time variation of the segregation efficiency, E at different air velocities, u. The results are given in Fig 4 of the main text.

5 Supplemental Video Legends

Video clips of the aeration of an initially well-mixed equal size glass beads and iron powder (~0.2 mm diameter) at 50:50 composition, over a period of 10 mins for air velocities between 6.2 cm/s and 14 cm/s played back at 20 time speed up.

6 References

1 Geldart, D. Types of Gas Fluidization. *Powder Technol*. **7**, 285 (1973).

2 Carman, P. C. Fluid flow through granular beds. *Trans. Inst. Chem. Eng. Lond.* **15**, 150 (1937).