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TENSILE STRESSES AROUND BOREHOLES DUE TO TRANSIENT FLUID FLOW

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SUMMARY

We present solutions for the effective stress induced by gas flow through a porous solid into a borehole resulting from sudden pressure reduction. Tensile effective stress that exceeds the strength of the solid will lead to borehole failure. This has applications to the intentional creation of cavities, relevant to the efficient recovery of coalbed methane, and the avoidance of borehole stability problems in conventional gas production.

1. INTRODUCTION

Borehole failure can result from several mechanisms.[1] Hydraulic fracture can be initiated by high pressures within the borehole.[2] Wellbore breakouts are attributed to high horizontal stress at depth.[3] These types of failure occur without consideration of the fluid flow *within the rock* itself. Fluid flow in a porous solid can also contribute to the state of stress in the solid.

The stress and failure induced by the flow of liquids into boreholes has been studied by Pasley and Cheatham[4] and Risnes *et al.*[5] The stress induced by gas flow does not seem to have received the same attention apart from brief mentions (e.g. Reference 6). Here we investigate the stresses induced by transient flow of gases into boreholes, and we make some comparisons to liquid flow. Gas is highly compressible, which provides pressure and stress profiles significantly different from those that occur with liquids. The compressibility of gas also means that the equation to be solved to describe the flow is non-linear, unlike the linear equation that describes the low-Reynolds-number flow of an incompressible liquid. It is this difficulty that has restricted solutions for gas flow to small pressure changes or other restrictive conditions so that linear approximations can be made. Failure of an underground opening is often associated with dramatic changes in pressure, such as the gas outbursts which occur in coal mines,[7] and the analogous laboratory experiments of Ujihira *et al.*[8] In these situations the non-linearity of the gas flow equation cannot be ignored.

A knowledge of the pressure profile allows the determination of the effective stress[9] on the porous solid. This may be useful not only for preventing or avoiding failure, but also on the occasions where failure is desired. This occasion arises in the creation of cavities for the exploitation of methane occurring naturally in coal. Specifically, Logan *et al.*[10] give a description of a coalbed methane completion technology called 'openhole cavity completion'. In this technology, a coalbed methane well is shut-in so that the pressure in the well approaches the original formation pressure of the coal seam. Then the well is suddenly opened to the atmosphere,

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resulting in a rapid pressure drop in the well. Coal has been observed to slough into the well on application of this procedure.

In this paper, we obtain numerical solutions for the effective stress in the porous medium for a model of the openhole cavity completion process, to estimate the characteristic distance and time scales associated with tensile failure of the medium in the vicinity of the borehole. For the openhole cavity completion process the physically interesting time scale (associated with the escape of gas into the borehole) is short. It therefore suffices here to model the porous medium by a linearly elastic continuum. There has been considerable interest in plastic deformation of wellbore surrounds (see, e.g. Geertsma[11] and references therein), but this issue is not relevant to the problem at hand, since the associated time scales far exceed those studied here.

2. STRESS AROUND A BOREHOLE

As in Geertsma,[14] we assume the system to be in a state of plane strain. As a result of tectonic forces, in the absence of any borehole, the radial and transverse principal stresses are uniform:

$$\sigma_{rr} = \sigma_{\theta\theta} = -\sigma_{\infty} \tag{2.1}$$

where σ_{∞} is a positive scalar. We have adopted the convention that the *normal stresses are positive in tension.* Stress increases at approximately 25 kPa m^{-1}, so that if we examine a layer several metres thick and 400 m below the surface, σ_{∞} will be of the order of 10 MPa. Suppose a vertical borehole of radius a is then drilled through the rock. Equation (2.1) will now hold as a boundary condition far from the borehole, but near the borehole the stress tensor will be a radially symmetric function of the distance from the axis of the borehole.

Whatever the constitutive properties of the rock, the stress tensor σ must satisfy the equilibrium equation

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0} \tag{2.2}$$

For plane strain with radial symmetry about the vertical (z) axis, free from shear, the equilibrium condition is satisfied if

$$\frac{\partial}{\partial r}\{r\sigma_{rr}\} - \sigma_{\theta\theta} = 0 \tag{2.3}$$

where r, θ and z are the usual polar co-ordinates.

We consider a homogeneous, isotropic, linearly elastic porous medium. In the absence of fluid, the constitutive equation for such a material relating the stress tensor $\boldsymbol{\sigma}$ to the strain tensor $\mathbf{e}$ is

$$\boldsymbol{\sigma} = \lambda_{\mathrm{L}} \operatorname{trace}\{\mathbf{e}\}\mathbf{I} + 2\mu_{\mathrm{L}}\mathbf{e} \tag{2.4}$$

where λ_{L} and μ_{L} are the Lamé constants and $\mathbf{I}$ is the unit tensor. The constitutive equation must be modified to account for the presence of the fluid, which can relieve the effective compressive stress on the matrix or even (as we show) produce a tensile effective stress. We use the modification proposed by Biot,[12,13] writing for the *total stress* $\boldsymbol{\sigma}$,

$$\boldsymbol{\sigma} = -\alpha p\mathbf{I} + \{\lambda_{\mathrm{L}} - \alpha^2 M\} \operatorname{trace}\{\mathbf{e}\}\mathbf{I} + 2\mu_{\mathrm{L}}\mathbf{e} \tag{2.5}$$

where α and M are parameters of the system which may be interpreted in terms of the physical model which leads to equation (2.5) and p is the fluid pressure. Briefly, the parameters α (dimensionless) and M (with dimensions of pressure) enter as coefficients in an expression for the strain energy as a quadratic function of the strain tensor and an appropriate measure of the increment of fluid content.[13]

For a purely radial displacement field $\mathbf{u}(r) = u(r)\hat{\mathbf{r}}$, the only non-zero components of the strain tensor are

$$e_{rr} = \frac{\mathrm{d}u}{\mathrm{d}r} \quad \text{and} \quad e_{\theta\theta} = \frac{u}{r} \tag{2.6}$$

and we have

$$\operatorname{trace}\{\mathbf{e}\} = \frac{\mathrm{d}u}{\mathrm{d}r} + \frac{u}{r} \tag{2.7}$$

(We have chosen to subtract off the uniform strain e_{zz} corresponding to the existing deformation of the porous medium before the fluid flow process takes place.) Equations (2.3), (2.5)–(2.7) lead to an equation for the radial displacement field:

$$u''(r) + \frac{u'(r)}{r} - \frac{u(r)}{r^2} = Kp'(r) \tag{2.8}$$

For brevity we have written

$$K = \alpha/(\lambda_{\mathrm{L}} + 2\mu_{\mathrm{L}} - \alpha^2 M) \tag{2.9}$$

It is easily verified that the general solution of this differential equation is

$$u(r) = \frac{K}{r}\int_a^r Rp(R)\,\mathrm{d}R + Ar + \frac{B}{r} \tag{2.10}$$

and we obtain the simple result that

$$\operatorname{trace}\{\mathbf{e}\} = Kp(r) + 2A \tag{2.11}$$

The solution for the stresses equivalent to equation (2.10) has been given previously by Geertsma.[14] In particular, we find after a little algebra (in which a number of terms involving the pressure $p(r)$ cancel) that

$$\sigma_{rr} = 2A\{\lambda_{\mathrm{L}} - \alpha^2 M\} + 2\mu_{\mathrm{L}}\left\{-\frac{K}{r^2}\int_a^r Rp(R)\mathrm{d}R + A - \frac{B}{r^2}\right\} \tag{2.12}$$

The arbitrary constants A and B are determined from boundary conditions at $r = a$ (the wellbore) and at $r = \infty$. These boundary conditions are most easily stated in terms of stresses.

At the wellbore, where the outward normal to the solid is $-\hat{\mathbf{r}}$, the solid is in contact with the atmosphere at ambient atmospheric pressure p_a, say, and we have the boundary condition

$$-\hat{\mathbf{r}}\cdot\boldsymbol{\sigma} = p_a\hat{\mathbf{r}} \tag{2.13}$$

so that

$$\sigma_{rr} = -p_a \quad \text{and} \quad \sigma_{r\theta} = 0 \tag{2.14}$$

The latter condition is automatically satisfied, but the former enables us to determine one of the constants in equation (2.12) and our solutions for the radial stress becomes

$$\sigma_{rr} = -p_a - \frac{2K\mu_{\mathrm{L}}}{r^2}\int_a^r Rp(R)\,\mathrm{d}R + \frac{2B}{a^2}\left\{1 - \frac{a^2}{r^2}\right\} \tag{2.15}$$

We assume an isotropic compressive state as $r \to \infty$. If we have [in accord with equation (2.1)] $\boldsymbol{\sigma} \to -\sigma_\infty\mathbf{I}$ and $p \to p_\infty$, the constant B is easily identified and we arrive at the key result, needed for later analysis, that

$$\sigma_{rr} = -p_a - 2Kp_\infty\mu_L\Pi(r) + \{Kp_\infty\mu_L - \sigma_\infty + p_a\}\left\{1 - \frac{a^2}{r^2}\right\} \tag{2.16}$$

where we have introduced the dimensionless quantity

$$\Pi(r) = \frac{1}{p_\infty r^2}\int_a^r Rp(R)\,\mathrm{d}R, \tag{2.17}$$

which converges to 1/2 as $r \to \infty$. The only way in which the elastic properties of the medium embodied in the parameters of Biot's constitutive equation enter the formula (2.16) for the stress is via the dimensionless quantity $2K\mu_L$, which Geertsma[14] notes is typically between 0·4 and 0·7 for rock. In the sample calculations reported below, we have taken $2K\mu_L = 0{\cdot}5$.

The total stress σ can be regarded as a linear combination of an *effective stress* on the solid σ^{eff} and the fluid stress $-p\mathbf{I}$:

$$\sigma = \sigma^{\text{eff}} - p\mathbf{I} \tag{2.18}$$

In particular, the radial component of the effective stress is

$$\sigma_{rr}^{\text{eff}} = \sigma_{rr} + p \tag{2.19}$$

If $\sigma^{\text{eff}} > 0$, the porous rock is effectively in tension, even though the total stress on the system is compressive. If the porous rock has a tensile strength $T \geqslant 0$ (in the absence of gas) tensile failure in the gas-filled rock will occur at any point where

$$\sigma^{\text{eff}} > T \tag{2.20}$$

In coal, the tensile strength is often very low.

In a more detailed analysis which follows, the pressure is time-dependent. In principle, for time-dependent problems, one should solve the time-dependent form of the momentum equation rather than the steady equilibrium condition (2.2), but we shall not address this here.

3. MODELS FOR FLUID FLOW

Water flow

Consider saturated liquid flow in a compressible aquifer, so that the fluid is taken as incompressible, while the pore space has a compressibility defined in Reference 14

$$\alpha_p = \frac{1}{\phi}\frac{\partial\phi}{\partial p} \tag{3.1}$$

where ϕ is the porosity. The continuity equation

$$\phi\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{q}) = 0 \tag{3.2}$$

relates the superficial velocity or volume flux $\mathbf{q}$ to the mass density ρ, while Darcy's law

$$\mathbf{q} = -\frac{k}{\mu}\nabla p \tag{3.3}$$

couples the superficial velocity to the pressure p in the liquid. The constants k and μ are, respectively, the permeability of the porous material and the shear viscosity of the liquid. In equation (3.3), we have omitted the effect of gravity, which for the systems to be modelled is unimportant. If we assume that the permeability and shear viscosity are constant, and that the pore compressibility is small and constant, we arrive at a diffusion equation for the pressure:

$$\frac{\partial p}{\partial t} = \frac{k}{\phi \mu \alpha_p} \nabla^2 p \tag{3.4}$$

Here, as usual, ∇^2 denotes the Laplace operator, and for circularly symmetric problems

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{3.5}$$

Gas flow

Gas flow in a porous rock is similar. Equations (3.2) and (3.3) remain valid, but as the compressibility of the gas far exceeds that of the rock, rather than using equation (3.1), we use the equation of state of the gas to eliminate the density in favour of the pressure. After a little algebra, we obtain the equation[15]

$$\frac{\partial \psi}{\partial t} = \frac{k}{\mu \phi c(\psi)} \nabla^2 \psi \tag{3.6}$$

where ψ is the pseudo-pressure, given by an integral involving the pressure and c is the gas compressibility. For convenience, we choose to consider isothermal flow of an ideal gas, so that the density is a linear function of pressure and

$$c(\psi) = 1/p, \qquad \psi(p) = p^2/\mu \tag{3.7}$$

Equation (3.6) thus reduces to the non-linear diffusion equation

$$\frac{\partial p}{\partial t} = \frac{k}{2\phi\mu} \nabla^2 (p^2) \tag{3.8}$$

In a previous paper,[15] we have given a numerical scheme for solving equation (3.6) and discussed analytic and empirical approximate solutions.

4. INSTANTANEOUS PRESSURE REDUCTON IN A BOREHOLE

Let us consider the case in which the gas pressure is initially p_∞, with the well pressure abruptly lowered from p_∞ to p_a and restrict our attention to such short times that no significant gas flow has occurred, so that as a first approximation we may replace p by p_∞ everywhere in $r > a$. Then in equation (2.16), we have $\Pi(r) \approx 1/2$ and

$$\sigma_{rr}^{\text{eff}} = p_\infty - p_a - \sigma_\infty - (K\mu_{\text{L}} p_\infty - \sigma_\infty + p_a)\frac{a^2}{r^2} \tag{4.1}$$

Since $\sigma_\infty > p_\infty > p_\infty - p_a$, so long as the inequality

$$K\mu_{\text{L}} < 1 \tag{4.2}$$

is satisfied, the effective stress is positive (i.e. tensile) in the region $a \leqslant r < r_0$, where

$$r_0/a = \left(\frac{\sigma_\infty - K\mu_{\text{L}} p_\infty + p_a}{\sigma_\infty - p_\infty + p_a}\right)^{1/2} \tag{4.3}$$

i.e. there is initially a region in tension close to the borehole. For typical coal reservoirs, one may take $\sigma_\infty \approx 4$ MPa, $p_\infty \approx 2$ MPa, $p_a \approx 0$, so that $r_0/a \approx \sqrt{(7/4)} \approx 1{\cdot}3$.

One would like to know for how long the tensile region endures. A reasonable estimate is the time τ taken for the pressure at radius r_0 to drop significantly. We estimate τ, assuming that $r_0/a \approx \sqrt{(7/4)}$ for a borehole of radius $a = 0{\cdot}12$ m in a medium of porosity $\phi = 0{\cdot}05$ and permeability $k = 1 \times 10^{-15}$ m^2 (1 md).

For *water*, an estimate of this time τ can be obtained from inspection of equation (3.4):

$$\tau = \frac{r_0^2 \phi \mu \alpha_p}{k} \tag{4.4}$$

Taking $\mu = 1 \times 10^{-3}$ Pa s [1 cp] and $\alpha_p = 4 \times 10^{-10}$ Pa^{-1}, we obtain $\tau \approx 0{\cdot}5$ s.

For *gas*, repeating the analysis above, using equation (3.8) we have

$$\tau = \frac{r_0^2 \phi \mu}{k p_\infty} \tag{4.5}$$

Assuming that $\mu = 1{\cdot}5 \times 10^{-5}$ Pa s [0·015 cp] and $p_\infty = 2$ MPa gives $\tau \approx 9{\cdot}5$ s.

In practice, the pressure in the wellbore cannot be dropped instantaneously to zero. For the illustrated example, there will be no prospect for tensile failure of the well unless the pressure can be reduced from 2 MPa to atmospheric pressure ($\approx 0{\cdot}1$ MPa) in time less than τ. Thus, the pressure drop in the well must be *extremely* fast ($< 0{\cdot}5$ s) for water-filled porous media. For gas, the pressure drop in the well must still be fast (< 10 s), but not as fast as for water. Moreover, in practical field applications, it is easier to reduce gas pressure rapidly by venting than to reduce water pressure by lifting water out of the borehole.

5. NUMERICAL SOLUTIONS

In an earlier paper[15] we have analyzed the transient flow of an ideal gas into a cylindrical borehole. The borehole has radius a and is initially at pressure p_∞. At time $t = 0$, the pressure in the borehole is instantaneously dropped to zero. This is an idealization and represents the extreme case. In the field, it will take some time to draw the pressure down to atmospheric pressure, which we have assumed so small compared to the initial pressure in the coal that it may be taken as zero. For short times, the numerical solutions for the pressure p at distance r from the centre of the borehole and time t in the problem are well fitted by the empirical curve[15]

$$(p/p_\infty)^2 = 1 - e^{-(0{\cdot}776\zeta + 0{\cdot}152\zeta^2)} \tag{5.1}$$

where

$$\zeta = (r/a - 1)(t_0/t)^{1/2} \tag{5.2}$$

and

$$t_0 = \frac{\phi \mu a^2}{k p_\infty} \tag{5.3}$$

In Figure 1 we compare the empirical pressure curve with our numerical solutions. In our numerical studies below, we use the empirical curve for $t \leqslant t_0$. For $t \leqslant 0{\cdot}1 t_0$ the empirical curve is the practical solution of the gas pressure problem and the effective stress calculations are accurate (within the approximations of the model and for the physical parameters chosen). We believe that for $0{\cdot}1 t_0 \leqslant t \leqslant t_0$, the results (though less accurate) will be a useful first approximation. For methane ($\mu = 1{\cdot}5 \times 10^{-5}$ Pa s, $p_\infty \approx 2$ MPa) leaking into a typical wellbore ($a \approx 0{\cdot}12$ m) from

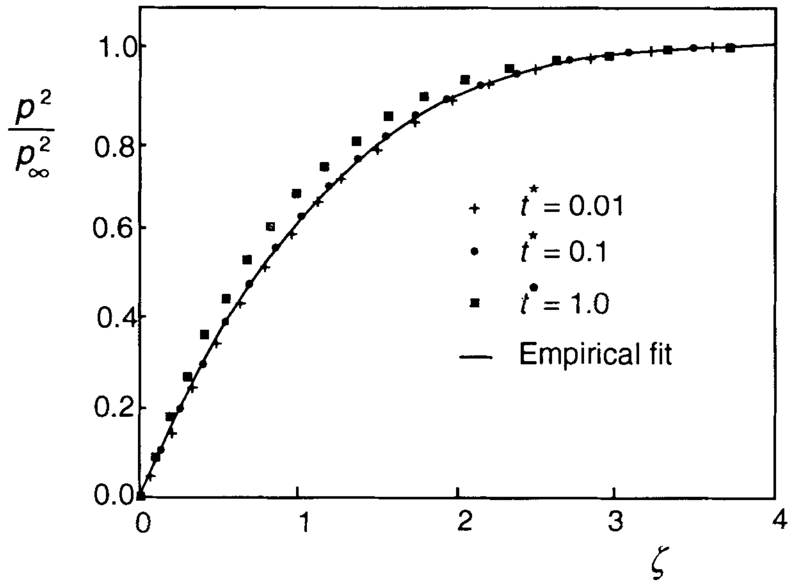

Figure 1. Radial flow of an ideal gas into a cylindrical cavity of radius a. The pressure p is subject to the boundary conditions that $p \to 0$ as $r \to a$ and $p \to p_\infty$ as $r \to \infty$. Here ζ and τ^* are as defined in equations (5.2) and (5.7). The numerical data and the empirical fit are from our earlier work (Reference 15).

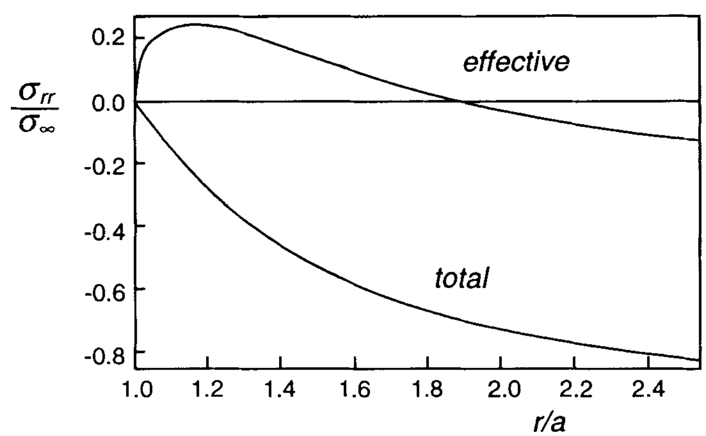

Figure 2. The total radial stress σ_{rr} and the effective radial stress σ_{rr}^{eff} in the case $p_\infty/\sigma_\infty = 0{\cdot}7$ at time $t/t_0 = 0{\cdot}05$. The tensile region is where $\sigma_{rr}^{\text{eff}} > 0$.

a coal seam ($k = 10^{-15}$ m^2[1 md], $\phi \approx 0{\cdot}05$), the characteristic time is

$$t_0 \approx 5{\cdot}2 \text{ s} \tag{5.4}$$

If we increase a to mineshaft diameters ($a \approx 1$ m), t_0 is increased to 6 min.

We shall use the empirical solution for the pressure to estimate for how long the effective stress is extreme enough that tensile failure may occur. This enables us to set an upper limit on the time interval in which the well-bore pressure must be reduced from p_∞ to atmospheric pressure if fracture of the coal surrounding the well-bore is to be possible. For more accurate computations,

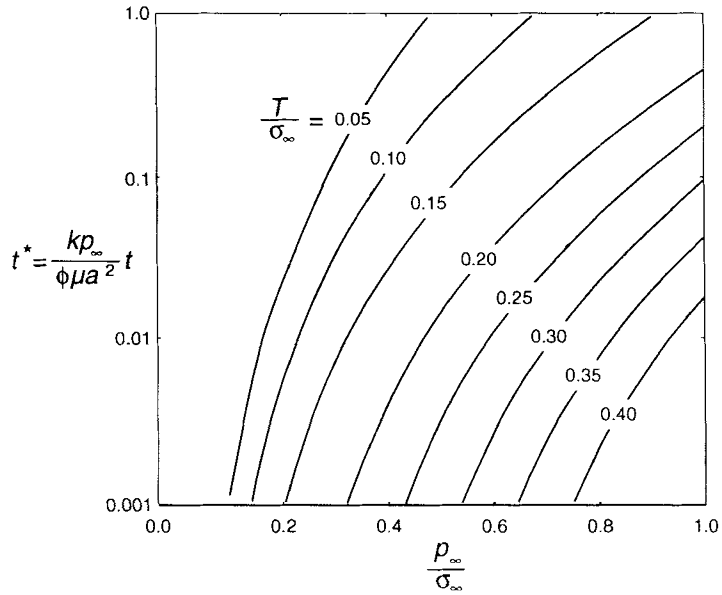

Figure 3. Numerical solutions for the period of time in which the effective radial stress σ_{rr}^{eff} exceeds the tensile stress T of the material.

especially at later times, the empirical solution (5.1) may be replaced by numerical solutions, using the implicit finite difference procedure described in our previous paper.[15] The numerical scheme is easily adapted to accommodate more general equations of state, and can also be generalized to wellbores or cavities which are not circular in cross-section.

With the empirical pressure curve (5.1), using equation (2.16) and setting $p_a = 0$, the effective radial stress can be written in the form

$$\frac{\sigma_{rr}^{\text{eff}}}{\sigma_\infty} = -2K\mu_{\text{L}}\frac{p_\infty}{\sigma_\infty}\Pi(r) + \left\{K\mu_{\text{L}}\frac{p_\infty}{\sigma_\infty} - 1\right\}\left\{1 - \frac{a^2}{r^2}\right\} \tag{5.5}$$

where

$$\Pi(r) = \frac{1}{r^{*2}}\int_a^{r^*} x\sqrt{\{1 - \exp[-0{\cdot}776(x-1)/t^{*1/2} - 0{\cdot}152(x-1)^2/t^*]\}}\,\mathrm{d}x \tag{5.6}$$

and we have introduced the dimensionless variables

$$r^* = r/a \quad \text{and} \quad t^* = t/t_0 \tag{5.7}$$

In Figure 2 we show the effective and the total radial stress as functions of position at one time. Failure will occur provided that the effective radial stress is tensile and of magnitude exceeding T. At small times, there is a big range in large tension region which promotes fracture, but the width of this region decreases with time.

We seek the time at which the region in adequate tension shrinks to a single value of r. At this time, the extreme value of the effective radial stress will coincide with the tensile strength. Our results are summarized in Figure 3, where we show the dimensionless time $t^* = t/t_0$ at which the tensile region vanishes as a function of p_∞/σ_∞ (the ratio of the gas pressure to the total stress far from the borehole). Each curve corresponds to a particular value of T/σ_∞. Examining this figure we can study particular cases. For example, consider the case when methane ($\mu \cong 1{\cdot}5 \times 10^{-5}$ Pa s) at initial pressure $p_1 = 2$ MPa, is suddenly released into a wellbore from porous rock with permeability $k = 10^{-15}$ m^2 (1 md), porosity $\phi = 0{\cdot}05$ and tensile strength 0·1 MPa in a stress field of 5 MPa. Then effective tension exceeding the tensile strength will exist for 4·7 s for a borehole of radius $a = 0{\cdot}12$ m. If the tensile strength is less, say 0·05 MPa, then a much longer period of 16·5 s exists. The possibility of failure depends critically on the borehole radius, with the time in tension increasing as the square of the borehole radius. The tensile strength of the porous rock is also important, with longer times in tension for weaker materials such a coal.

These values serve to provide design parameters. If one is attempting to create a cavity, then it would often be necessary to reduce the borehole pressure in the order of a few seconds. Alternatively, if one were trying to avoid borehole failure by this mechanism, then rapid reductions in pressure should be avoided.

6. CONCLUSIONS

In this paper we have discussed the effective stress in the matrix of a porous solid due to fluid flow. For fluid draining into a borehole, transient tensile effective stresses can be generated in the vicinity of the borehole. If these tensile effective stresses exceed the tensile strength of the matrix, the borehole may fracture. Simple time-scale estimates suggest that for gas-filled porous rocks, reductions of borehole pressure in the order of a few seconds can lead to tensile failure of the borehole. The time scales for water-filled porous media are so short that it is difficult to achieve tensile failure of a borehole. For the case of gas flow, we have given more careful numerical estimates of the relevant time scales using our previously obtained empirical solutions for the pressure.

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