Communication

Efficient Field-Only Surface Integral Equations for Electromagnetics

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***Abstract*—In a recent paper, Klaseboer *et al.* (IEEE Trans. Antennas Propag., vol. 65, no. 2, pp. 972-977, Feb. 2017) developed a surface integral formulation of electromagnetics that does not require working with integral equations that have singular kernels. Instead of solving for the induced surface currents, the method involves surface integral solutions for 4 coupled Helmholtz equations: 3 for each Cartesian component of the electric $\boldsymbol{E}$ field plus 1 for the scalar function $(\boldsymbol{r}\cdot\boldsymbol{E})$ on the surface of scatterers. Here we improve on this approach by advancing a formulation due to Yuffa *et al.* (IEEE Trans.Antennas Propag., vol. 66, no. 10, pp. 5274-5281, Oct. 2018) that solves for $\boldsymbol{E}$ and its normal derivative. Apart from a 25% reduction in problem size, the normal derivative of the field is often of interest in micro-photonic applications.**

***Index Terms*—Maxwell equations, boundary element methods, boundary integral equations, electric and magnetic field integral equation, electromagnetic propagation and scattering, alternative electromagnetic theory, Helmholtz equations, vector wave equation.**

I. Introduction

There have been two recent independent developments in formulating computational electromagnetics in terms of surface integral equations that are conceptually very different from the venerable theoretical framework of Stratton–Chu that was established almost 80 years ago [1], [2]. Whereas the Stratton–Chu approach is based on solving for surface currents at boundaries, the recent works are based on solving directly for components of the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields. One of these field-only formulations had its genesis in the study of scattering from rough surfaces [3] some 25 years ago but has been generalized recently with extensive use of tensorial formalism [4], [5]. An independently developed field-only formulation focused on the use of non-singular surface integral equations for the field components and was motivated by the fact that theoretical representations of physical phenomena that are finite and well-behaved physically on boundaries should not require the use of theoretical models that contain mathematical singularities [6], [7].

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The aim of this communication is to harmonize these field-only formulations by using accessible Gibbsian vector notation to reveal the physical basis of the model. This approach turns out to be conceptually simple, provides direct access to values of field amplitude and field gradients at boundaries, is able to be implemented numerically using simple, efficient and accurate algorithms and reduced the problem size by 25% compared to earlier work [6], [7].

The detailed formulation is followed by a discussion of the computational advantages and applications and supported by numerical illustrations.

II. Formulation

We illustrate our electromagnetics formulation with the scattering of an incident plane wave by 3D perfect electrical conductors (PECs). In the frequency domain with time dependence $\exp(\mathrm{j}\omega t)$, the propagating scattered electric field $\boldsymbol{E}$ is given by the wave equation (in a source free region) $(k^2 = \omega^2\epsilon_0\epsilon_r\mu_0\mu_r \equiv \omega^2\epsilon\mu)$

$$\nabla^2\boldsymbol{E} + k^2\boldsymbol{E} = 0 \quad \text{with} \quad \nabla\cdot\boldsymbol{E} = 0. \tag{1}$$

These equations hold for the incident, $\boldsymbol{E}^{\text{inc}}$, and total $\boldsymbol{E}^{\text{tot}} = \boldsymbol{E}^{\text{inc}} + \boldsymbol{E}$ fields. Since $\nabla\cdot\boldsymbol{E} = 0$, there are only two independent components of $\boldsymbol{E}$ in (1) and they are found by specifying the incident wave, for example a plane wave $\boldsymbol{E}^{\text{inc}} = \boldsymbol{E}_0\exp(-\mathrm{j}\boldsymbol{k}\cdot\boldsymbol{r})$, where $\boldsymbol{r} = (x, y, z)$ is the position vector and imposing the boundary condition that the two independent tangential components of $\boldsymbol{E}^{\text{tot}}$ must vanish on the surface, S of the PEC.

In earlier work [6], [7] the condition $\nabla\cdot\boldsymbol{E} = 0$ was replaced using a vector identity by a Helmholtz equation for $(\boldsymbol{r}\cdot\boldsymbol{E})$:

$$2(\nabla\cdot\boldsymbol{E}) \equiv \nabla^2(\boldsymbol{r}\cdot\boldsymbol{E}) + k^2(\boldsymbol{r}\cdot\boldsymbol{E}) = 0. \tag{2}$$

This means the solution of (1) can be cast as the solution of 4 scalar Helmholtz equations: one for each of the 3 components of $\boldsymbol{E}$ and one for the function $(\boldsymbol{r}\cdot\boldsymbol{E})$.

The solution of the vector Helmholtz equation for $\boldsymbol{E}$ in (1) can be expressed as the solution of the surface integral equation for each Cartesian component of $\boldsymbol{E}$ $(i = x, y, z)$

$$c_0 E_i(\boldsymbol{r}_0) + \int_S E_i(\boldsymbol{r})\frac{\partial G(\boldsymbol{r},\boldsymbol{r}_0)}{\partial n}\, dS(\boldsymbol{r}) = \int_S \frac{\partial E_i(\boldsymbol{r})}{\partial n} G(\boldsymbol{r},\boldsymbol{r}_0)\, dS(\boldsymbol{r}) \tag{3}$$

where $G(\boldsymbol{r},\boldsymbol{r}_0) = \exp(-\mathrm{j}k|\boldsymbol{r}-\boldsymbol{r}_0|)/|\boldsymbol{r}-\boldsymbol{r}_0|$ is the Green's function and $\partial/\partial n \equiv \boldsymbol{n}\cdot\nabla$ with $\boldsymbol{n}$ being the unit normal that points out of the 3D domain into the scatterer. The integration point $\boldsymbol{r}$ is on the surface, S of the scatterer. If $\boldsymbol{r}_0$ is located outside the scatterer in the 3D domain, $c_0 = 4\pi$ but if r_0 is on the surface, S, then c_0 represents the solid angle subtended at $\boldsymbol{r}_0$ and is 2π provided that the tangent plane is continuous at $\boldsymbol{r}_0$.

Any derivative of each Cartesian component of $\boldsymbol{E}$ is also a solution of the Helmholtz equation, therefore it immediately follows that $\nabla \cdot \boldsymbol{E}$ also satisfies: $\nabla^2(\nabla \cdot \boldsymbol{E}) + k^2(\nabla \cdot \boldsymbol{E}) = 0$. Thus a similar surface integral equation as (3) is valid for $(\nabla \cdot \boldsymbol{E})$ as

$$\begin{aligned} c_0(\nabla \cdot \boldsymbol{E}(\boldsymbol{r}_0)) + \int_S (\nabla \cdot \boldsymbol{E}(\boldsymbol{r})) \frac{\partial G(\boldsymbol{r}, \boldsymbol{r}_0)}{\partial n}\ dS(\boldsymbol{r}) \\ = \int_S \frac{\partial(\nabla \cdot \boldsymbol{E}(\boldsymbol{r}))}{\partial n} G(\boldsymbol{r}, \boldsymbol{r}_0)\ dS(\boldsymbol{r}). \end{aligned} \tag{4}$$

Since $(\nabla \cdot \boldsymbol{E})$ vanishes on the surface, the left hand side of (4) is zero. Therefore $\partial(\nabla \cdot \boldsymbol{E})/\partial n$ is zero as well. Applying (4) off the surface (by setting $\boldsymbol{r}_0$ in the domain) will then ensure that $\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = 0$ in the entire 3D domain. The incoming undisturbed wave automatically satisfies $\nabla \cdot \boldsymbol{E}^{\text{inc}} = 0$.

The value of $(\nabla \cdot \boldsymbol{E})$ on the surface, or rather its limiting value as one approaches the surface with surface normal, $\boldsymbol{n}$ and surface tangent $\boldsymbol{t}$ can be found by first projecting $\boldsymbol{E}^{\text{tot}}$ onto the normal, $(\boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{n})$ and tangential, $(\boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{t})$ components, and noting that the latter $(\boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{t}) = 0$ on the PEC surface. This gives

$$\nabla \cdot \boldsymbol{E}^{\text{tot}} = \nabla \cdot \Big[\boldsymbol{n} \cdot (\boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{n})\Big] \tag{5a}$$

$$\begin{aligned} &= (\nabla \cdot \boldsymbol{n})(\boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{n}) + [(\boldsymbol{n} \cdot \nabla)\boldsymbol{E}^{\text{tot}}] \cdot \boldsymbol{n} \\ &\quad + \boldsymbol{E}^{\text{tot}} \cdot [(\boldsymbol{n} \cdot \nabla)\boldsymbol{n}] \end{aligned} \tag{5b}$$

From Appendix A, we see that the term $(\nabla \cdot \boldsymbol{n}) = -\kappa$, where κ is the sum of the principal curvatures of the scatterer and the normal derivative of the surface normal vector in the last term in (5b) is a zero vector: $(\boldsymbol{n} \cdot \nabla)\boldsymbol{n} = \boldsymbol{0}$. The condition $\nabla \cdot \boldsymbol{E}^{\text{tot}} = 0$ gives the following condition on the PEC surface [5]:

$$\kappa(\boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{n}) = [(\boldsymbol{n} \cdot \nabla)\boldsymbol{E}^{\text{tot}}] \cdot \boldsymbol{n} \tag{6a}$$

$$= \frac{\partial \boldsymbol{E}^{\text{tot}}}{\partial n} \cdot \boldsymbol{n} \tag{6b}$$

that is also a boundary condition on the scattered field: $\boldsymbol{E} = \boldsymbol{E}^{\text{tot}} - \boldsymbol{E}^{\text{inc}}$ involving the normal component of $\boldsymbol{E}$ and the normal component of the normal derivative of $\boldsymbol{E}$ on the surface of the PEC. This relation will also ensure that $\boldsymbol{E}$ is divergence free in the 3D domain as demonstrated above. The result (6) has a simple physical interpretation. From elementary electrostatics, the field emanating from a charged planar PEC is constant, that is $\partial \boldsymbol{E}^{\text{tot}}/\partial n = \boldsymbol{0}$ and is directed normal to the surface. In (6), $(\boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{n})$ is the induced charged density at the PEC surface and so $(\partial \boldsymbol{E}^{\text{tot}}/\partial n) \cdot \boldsymbol{n}$ can only be non-zero if the PEC has a non-zero curvature, κ.

In summary, the electric field due to scattering by PEC scatterers can be found by solving the 3 surface integral equation (3) for each of the Cartesian components of the scattered field, $\boldsymbol{E}$ and imposing the boundary condition (6) together with the requirement that the two tangential components of $\boldsymbol{E}^{\text{tot}}$ disappear on the boundary. These constitute the necessary and sufficient conditions to determine the scattered field, $\boldsymbol{E}$ and also ensure that $\boldsymbol{E}$ is divergence free as required.

Similarly, the magnetic field can be found by solving the surface integral equation corresponding to (3) for the scattered $\boldsymbol{H}$ field with the boundary condition $(\boldsymbol{H}^{\text{tot}} \cdot \boldsymbol{n}) = 0$ at the PEC surface. The boundary condition on the tangential components of $\boldsymbol{E}^{\text{tot}}$ was applied by choosing two orthogonal unit tangential vectors $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ on S, and using Ampere's law to express the component of $\boldsymbol{E}^{\text{tot}}$ parallel to say, $\boldsymbol{t}_1$, namely, $E^{\text{tot}}_{t1} \equiv \boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{t}_1 = \boldsymbol{E}^{\text{tot}} \cdot (\boldsymbol{t}_2 \times \boldsymbol{n})$, in terms of $\boldsymbol{H}^{\text{tot}}$ [6]

$$E^{\text{tot}}_{t1} = \boldsymbol{t}_2 \cdot (\boldsymbol{n} \times \boldsymbol{E}^{\text{tot}}) = \frac{1}{\text{j}\omega\epsilon}\{\boldsymbol{t}_2 \cdot (\boldsymbol{n} \times \nabla \times \boldsymbol{H}^{\text{tot}})\} \tag{7a}$$

$$= \frac{1}{\text{j}\omega\epsilon}\{\boldsymbol{n} \cdot (\boldsymbol{t}_2 \cdot \nabla)\boldsymbol{H}^{\text{tot}} - \boldsymbol{t}_2 \cdot (\boldsymbol{n} \cdot \nabla)\boldsymbol{H}^{\text{tot}}\} = 0 \tag{7b}$$

with a similar expression for $E^{\text{tot}}_2 \equiv \boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{t}_2$ obtained by interchanging subscripts $t1$ and $t2$ in (7).

Before presenting illustrative numerical results we consider the computational advantages and applications of the present formulation.

III. Computational advantages and applications

The Stratton–Chu [1], [2] surface integral was developed by the classic works of Poggio & Miller [8], Chang & Harrington [9] and Wu & Tsai [10] (PMCHWT) and is now one of the standard approaches to calculate frequency domain electromagnetics. In the PMCHWT formulation, the electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{H}$, are given in terms of electric and magnetic surface currents (or scalar and vector potentials [11]). The fields $\boldsymbol{E}$ and $\boldsymbol{H}$ can be then obtained by post-processing the surface current values. Many numerical methods have been developed to solve the surface current integral equations. An often used scheme employs the Rao-Wilton-Glisson (RWG) [12] basis functions which conserve charge (flat triangular surface elements are used to represent surface currents). This approach still has a lot of challenges [13].

Although it is also possible to develop vector surface integral equations for $\boldsymbol{E}$ and $\boldsymbol{H}$, such equations will involve hypersingular dyadic Green's functions as kernels [14] and introduce numerical difficulties in the zero frequency (long wavelength) limit [14], [15]. The strong singularities of the kernel that are inherent in this approach mean that it is challenging to obtain numerically accurate field values [16], let alone field gradient values near boundaries.

In contrast, the boundary integral solution of (3) for components of the field is conceptually straightforward. It is based on Green's Second Identity which provides a relation between $p(\boldsymbol{r})$ and its normal derivative $\partial p/\partial n$ at points $\boldsymbol{r}$ and $\boldsymbol{r}_0$ on the boundary, S. All singularities of the Green's function $G(\boldsymbol{r}, \boldsymbol{r}_0)$, can in fact be removed analytically to give [17], [18]

$$\begin{aligned} \int_S [p(\boldsymbol{r}) - p(\boldsymbol{r}_0)g(\boldsymbol{r}) - \frac{\partial p(\boldsymbol{r}_0)}{\partial n} f(\boldsymbol{r})] \frac{\partial G}{\partial n} dS(\boldsymbol{r}) = \\ \int_S G[\frac{\partial p(\boldsymbol{r})}{\partial n} - p(\boldsymbol{r}_0)\frac{\partial g(\boldsymbol{r})}{\partial n} - \frac{\partial p(\boldsymbol{r}_0)}{\partial n}\frac{\partial f(\boldsymbol{r})}{\partial n}] dS(\boldsymbol{r}). \end{aligned} \tag{8}$$

The functions $f(\boldsymbol{r})$ and $g(\boldsymbol{r})$ must satisfy the Helmholtz equation and the following requirements at $\boldsymbol{r} = \boldsymbol{r}_0$ on the

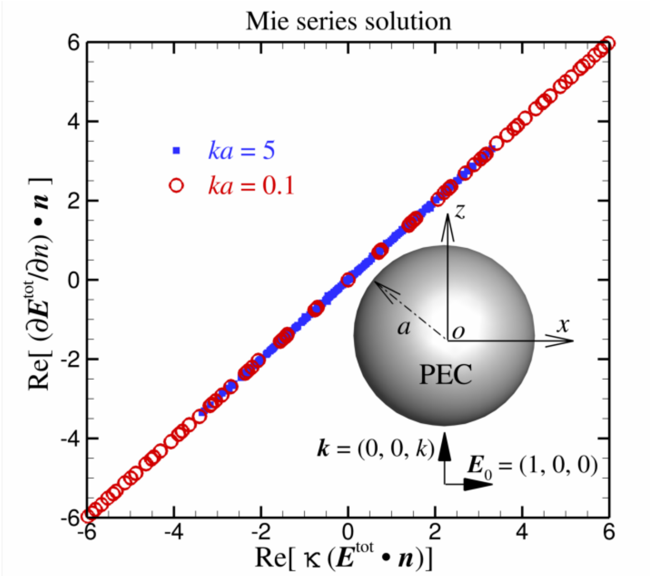

Fig. 1. A graphical confirmation of the boundary condition (6) that relates $(\boldsymbol{E}^{\text{tot}} \cdot \boldsymbol{n})$ and $(\partial \boldsymbol{E}^{\text{tot}}/\partial \boldsymbol{n}) \cdot \boldsymbol{n}$ that follows from enforcing $\nabla \cdot \boldsymbol{E}^{\text{tot}} = 0$ on the boundary. The values are obtained from the Mie series solution for the scattering of a plane wave by a PEC sphere with $ka = 0.1$ (open symbols) and 5 (solid symbols).

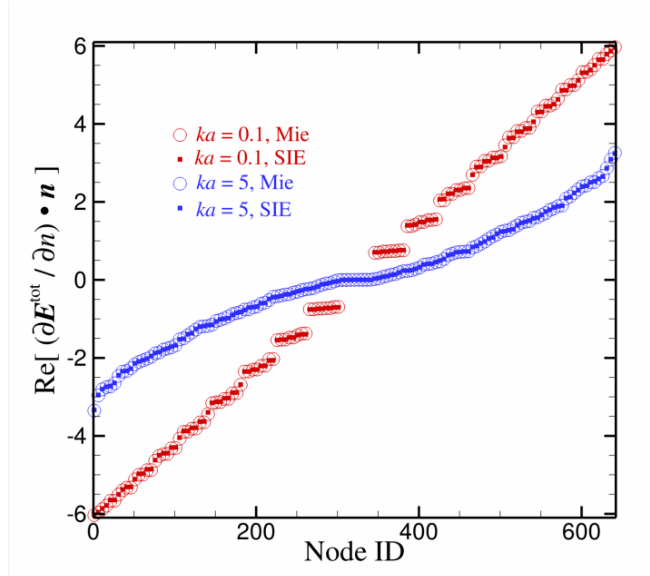

Fig. 2. Comparisons of the field gradient $(\partial \boldsymbol{E}^{\text{tot}}/\partial n) \cdot \boldsymbol{n}$ obtained from using the present surface integral implementation - SIE (solid symbols) and from the Mie series solution (open symbols) of scattering of an incident plane wave by a PEC sphere. A non-singular boundary integral method with 642 degrees of freedom being the unknowns on 642 nodes connecting 320 quadratic area elements that span the PEC sphere surface.

surface, S: $f(\boldsymbol{r}) = 0, \boldsymbol{n} \cdot \nabla f(\boldsymbol{r}) = 1, g(\boldsymbol{r}) = 1, \boldsymbol{n} \cdot \nabla g(\boldsymbol{r}) = 0$. Possible choices for $f(\boldsymbol{r})$ and $g(\boldsymbol{r})$ can be found in [17], [18]. Thus if p (or $\partial p/\partial n$) is given, then (8) can be solved for $\partial p/\partial n$ (or p) in a straightforward manner. The reason is that for $f(\boldsymbol{r})$ and $g(\boldsymbol{r})$ that obey the above mentioned conditions, the terms that multiply with G and $\partial G/\partial n$ vanish at the same rate as G or $\partial G/\partial n$ diverge when $\boldsymbol{r} \to \boldsymbol{r}_0$. Thus both integrals become non-singular and can thus be evaluated accurately by standard Gauss quadrature [17], [18]. The solid angle, c_0 at $\boldsymbol{r}_0$ has conveniently been eliminated in (8) as well.

In applications ranging from antenna metrology in which near field values are used to predict far field performance to micro-photonics in which accurate knowledge of values of the field and field gradients near surface is used to quantify effects due to surface enhanced Raman spectroscopy it is particularly advantageous to be able to work directly with values of the field and field gradient as in the present formulation. In our implementation, these quantities are unknowns to be solved at chosen nodes on the surface. In the evaluation of surface integrals, the surface shapes are taken as quadratic elements interpolated from the surface nodes and variations of function values within these elements are interpolated from the nodal values. Since the integrals do not have divergent kernels, the integration can be evaluated to high accuracy using standard Gauss quadrature. This facilitates the reduction of the number of degrees of freedom while increasing numerical precision. Also with high order surface elements, surface geometries can be represented more faithfully than with planar elements. Finally, with the help of (6), a matrix system is constructed, where the unknowns are $(\boldsymbol{E} \cdot \boldsymbol{n})$, and the two tangential components of $\partial \boldsymbol{E}/\partial n$ for each node. Further details of our implementation are given in [6].

IV. Illustrative numerical results

We provide illustrative numerical results of the present surface integral method for the scattering of an incident plane wave by a PEC sphere. The incident plane wave: $\boldsymbol{E}^{\text{inc}} = \boldsymbol{E}_0 \exp(-\mathrm{j}kz)$ is x-polarized and propagates along the z-direction. For a sphere of radius, a, the total curvature is $\kappa = 2/a$. We compare our results with the well-known analytical series solution for Mie scattering [19].

The results in Fig. 1 serve as graphical confirmations of the validity of the boundary condition (6) that is the consequence of enforcing $\nabla \cdot \boldsymbol{E}^{\text{tot}} = 0$ on the PEC boundary. The equality of the two sides of (6) for $ka = 0.1$ and 5, a 50-fold variation in ka at various points on the sphere is clearly evident, verifying (6) using a well-known analytical result. Note that the sphere with a smaller radius of curvature has larger range of magnitudes for the field gradient and surface charge.

To illustrate the ability of the present method to calculate accurately the field gradient $(\partial \boldsymbol{E}^{\text{tot}}/\partial n) \cdot \boldsymbol{n}$ we compare in Fig. 2, results obtained from the present surface integral method and that obtained from the Mie series solution for $ka = 0.1$ and 5. For visual clarity, the data presented in Fig. 2 have been ordered in increasing order of the value of $(\partial \boldsymbol{E}^{\text{tot}}/\partial n) \cdot \boldsymbol{n}$. When the value of $(\partial \boldsymbol{E}^{\text{tot}}/\partial n) \cdot \boldsymbol{n}$ is around 1, the relative difference between the result by the Mie series solution and that by the surface integral method is less than 1.6%. We note that the field gradient is one of the unknowns to be solved in the present surface integral formulation but in other surface integral implementations, it can only be obtained by post-processing, an additional computational step that might reduce numerical accuracy.

V. Conclusions

The field-only formulation of computational electromagnetics developed here has a number of meritorious features:

1) The formulation is simple conceptually and only encompasses the key physical tasks of computational electromagnetics, namely, solving the vector wave equation with the divergence free constraint without complication.
2) Physically important values of the field and its derivative at the surface are obtained directly without the need to work with intermediate quantities such as the surface current.
3) Only solutions of scalar Helmholtz equations are required and an accurate non-singular surface integral method is readily available. The absence of singular kernels facilitates the use of high order area elements that provides a more precise representation of surface geometries. The reliance on only finding solution of scalar Helmholtz equations may also be advantageous in solving time-domain scattering problems using an inverse Fourier Transform [20].
4) Numerical challenges such as singular or hypersingular integrals [14] and the zero frequency catastrophe [15] that preclude the accurate evaluation of field quantities at or near a surface are consequences of the mathematical formulation. The present approach is not affected by such issues.

Appendix A
Results from differential geometry

We present a simple derivation of the results from differential geometry used in (5) and (6). Erect a local Cartesian system with the origin at the point $\boldsymbol{r}_0$ on the surface, S and constant unit vectors: $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. The normal at $\boldsymbol{r}_0$ is chosen to be the $\mathbf{k}$ direction with surface tangents along the $\mathbf{i}$ and $\mathbf{j}$ directions. We assume quite generally that the surface around $\boldsymbol{r}_0$ is locally quadratic so that the coordinates of points $\boldsymbol{r} = (\xi_1, \xi_2, \zeta)$ that lie on the surface near $\boldsymbol{r}_0$ obey the relation

$$\Phi(\xi_1, \xi_2, \zeta) \equiv \zeta - \tfrac{1}{2}\kappa_1 \xi_1^2 - \tfrac{1}{2}\kappa_2 \xi_2^2 = 0 \tag{9}$$

with constants κ_1 and κ_2 being the principal curvatures at $\boldsymbol{r}_0$. With the gradient operator in local coordinates given by

$$\nabla = \mathbf{i}\frac{\partial}{\partial \xi_1} + \mathbf{j}\frac{\partial}{\partial \xi_2} + \mathbf{k}\frac{\partial}{\partial \zeta} \tag{10}$$

the unit normal at $\boldsymbol{r}$ is

$$\boldsymbol{n} = \frac{\nabla\Phi}{|\nabla\Phi|} = \frac{(-\kappa_1\xi_1, -\kappa_2\xi_2, 1)}{[1 + (\kappa_1\xi_1)^2 + (\kappa_2\xi_2)^2]^{1/2}}. \tag{11}$$

Thus it follows from the above that at $\boldsymbol{r} = \boldsymbol{r}_0 = (0,0,0)$

$$\nabla \cdot \boldsymbol{n} = -(\kappa_1 + \kappa_2) \equiv -\kappa, \tag{12}$$

and

$$(\boldsymbol{n} \cdot \nabla)\boldsymbol{n} = \mathbf{0}. \tag{13}$$

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