Adjoint Sensitivity Analysis for Attribution of Responsibility for Climate Change

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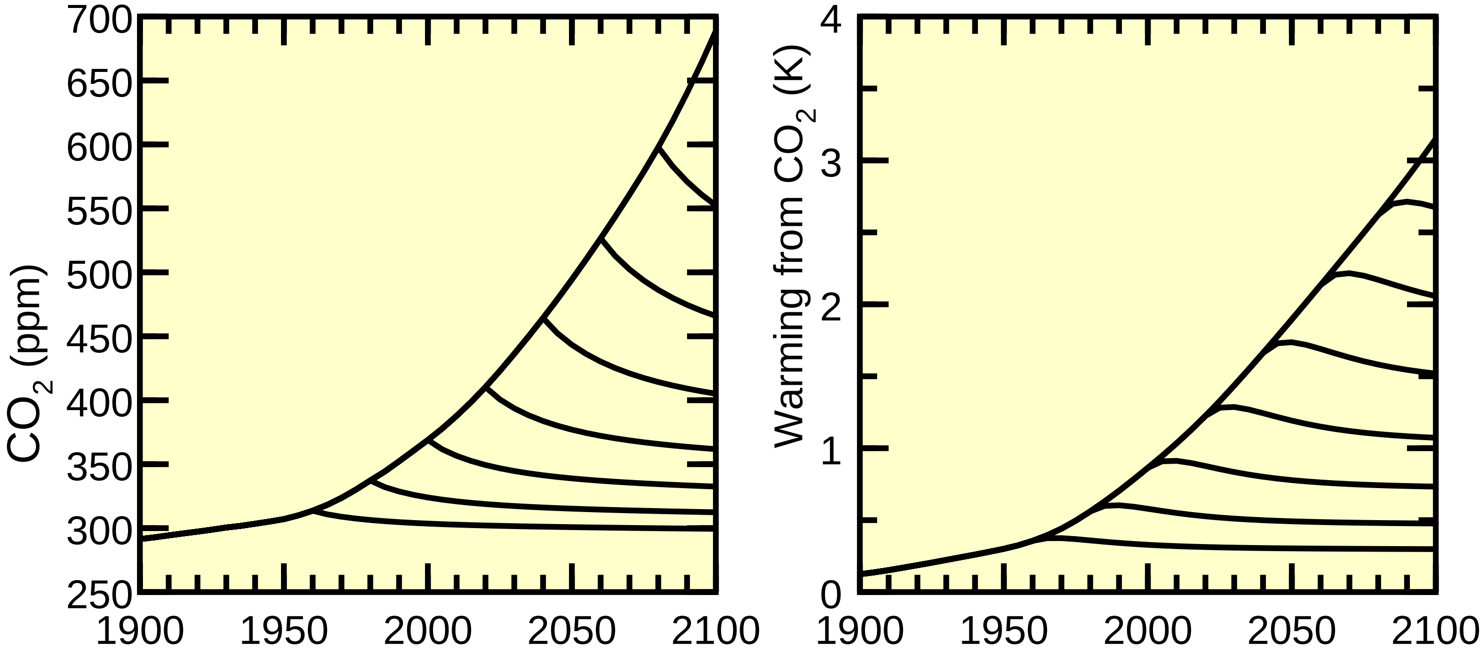
Summary

- Time-scales for the greenhouse effect
  – committed warming
- The Brazilian Proposal
  – setting reduction targets in proportion to responsibility
- Adjoint modelling
  – efficient calculation of sensitivities
- Analysing the Brazilian proposal
  – Who's to blame for the greenhouse effect?

Timescales

CO_2 concentrations and consequent warming, partitioned according to time of emission.



Lowest bands are from pre-1960 emissions, next from 1960 to 1980 emissions, etc. Increase in contribution to warming after time of emissions from ‘committed warming’ effect.

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Brazilian Proposal

Tabled by Brazil during negotiations leading to Kyoto Protocol — Flicked-passed to Subsidiary Body for Scientific and Technical Advice (SBSTA).

Proposes that emission reduction targets should be proportional to nations' relative responsibility for the greenhouse effect.

Issues:

- Indicator? What quantity is used as a measure of the greenhouse effect?
- For what period of emissions is responsibility attributed?
- How are non-linear responses attributed?

Brazilian Proposal as Derivatives

As example, use indicator $T^* = T_{\mathrm{CO2}}(2000)=$ warming in 2000 from CO_2 emissions.

T^* is to be attributed to emissions $E_j(t)$ from country j with $E(t) = \sum_j E_j(t)$.

Differential attribution to country j of emissions at time t is

$$\frac{\partial T^*}{\partial E_j(t)} E_j(t) = \frac{\partial T^*}{\partial E(t)} E_j(t) = S(t) E_j(t)$$

where $S(t)$ is a Fréchet derivative.

Cumulated attribution: $T_j^* = \int S(t) E_j(t)\, dt$

Aims of adjoint modelling

Aim is to simplify calculations by separating parametric differentiation from model integration, expressed here in terms of Green's function $\mathcal{G}$ of $\mathcal{L}\underline{u}(.) = \underline{f}(.)[\underline{a}]$.

Considers $\nabla_{\underline{a}}\langle \underline{w}(.)|\underline{u}(.)[\underline{a}]\rangle$
where $\underline{u}(.)[\underline{a}] = \mathcal{G}\underline{f}(.)[\underline{a}]$ (or $\mathcal{L}\underline{u}(.) = \underline{f}(.)[\underline{a}]$)

Transforms as
$\nabla_{\underline{a}}\langle \underline{w}(.)|\underline{u}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle \underline{w}(.)|\mathcal{G}\underline{f}(.)[\underline{a}]\rangle =$
$\nabla_{\underline{a}}\langle \mathcal{G}^{\dagger}\underline{w}(.)|\underline{f}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle \underline{v}(.)|\underline{f}(.)[\underline{a}]\rangle$
where $\underline{v}(.) = \mathcal{G}^{\dagger}\underline{w}(.)$ defines a single function $\underline{v}(.)$ with no dependence of $\underline{a}$.

Principles of adjoint modelling

Given $\underline{u}(.)[\underline{a}] = \mathcal{G}\underline{f}(.)[\underline{a}]$, where typically $\mathcal{L}\underline{u}(.)$ is linearisation of a more general model:

Formally:

$$\nabla_{\underline{a}}\langle\underline{w}(.)|\underline{u}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle\underline{w}(.)|\mathcal{G}\underline{f}(.)[\underline{a}]\rangle =$$
$$\nabla_{\underline{a}}\langle\mathcal{G}^{\dagger}\underline{w}(.)|\underline{f}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle\underline{v}(.)|\underline{f}(.)[\underline{a}]\rangle$$

with $\underline{v}(.) = \mathcal{G}^{\dagger}\underline{w}(.)$

In practice, used as $\mathcal{L}\underline{u}(.) = \underline{f}(.)[\underline{a}]$)

$$\nabla_{\underline{a}}\langle\underline{w}(.)|\underline{u}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle\mathcal{L}^{\dagger}\underline{v}(.)|\underline{u}(.)[\underline{a}]\rangle =$$
$$\nabla_{\underline{a}}\langle\underline{v}(.)|\mathcal{L}\underline{u}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle\underline{v}(.)|\underline{f}(.)[\underline{a}]\rangle$$

with $\underline{w}(.) = \mathcal{L}^{\dagger}\underline{v}(.)$ giving equations for adjoint model.

Applying adjoint modelling

Differentiation (only case used in this talk)

$$\nabla_{\underline{a}}\langle \underline{w}(.)|\mathcal{G}\underline{f}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle \mathcal{G}^{\dagger}\underline{w}(.)|\underline{f}(.)[\underline{a}]\rangle$$

Gradients for soft constraints .

$$\nabla_{\underline{a}}\langle \underline{Hu} - \underline{z}|\underline{Hu} - \underline{z}\rangle = 2\nabla_{\underline{a}}\langle \underline{Hu}_0 - \underline{z}|\underline{Hu}\rangle =$$
$$2\nabla_{\underline{a}}\langle \underline{Hu}_0 - \underline{z}|\underline{H}\mathcal{L}\underline{f}\rangle = 2\nabla_{\underline{a}}\langle (\underline{H}\mathcal{L})^{\dagger}(\underline{Hu}_0 - \underline{z})|\underline{f}\rangle$$

Gradients, with hard constraints: $\mathcal{L}\underline{u}(.) = 0$

$$\Theta^{*} = \Theta - \langle \underline{v}(.)|\mathcal{L}\underline{u}(.)\rangle$$

The function $\underline{v}(.)$ is the Lagrange multiplier.

$\nabla_{\underline{u}}\Theta^{*} = \nabla_{\underline{u}}\Theta - \nabla_{\underline{u}}\langle \mathcal{L}^{\dagger}|\underline{u}(.)\rangle$, whence

$\mathcal{L}^{\dagger}\underline{v}(.) = \nabla_{\underline{u}}\Theta$ — the adjoint equations define the Lagrange multiplier

Tangent Linear Model (TLM)

For N DEs: $\quad \dfrac{d}{dt}x_j = g_j(\{x_k\}, a, t) \qquad$ for $j = 1, N$

we can define sensitivities as

$$y_j = \frac{\partial}{\partial a}x_j \qquad \text{for } j = 1, N \text{ or} \qquad y_{j,p} = \frac{\partial}{\partial a_p}x_j$$

to give 'tangent linear model(s)':

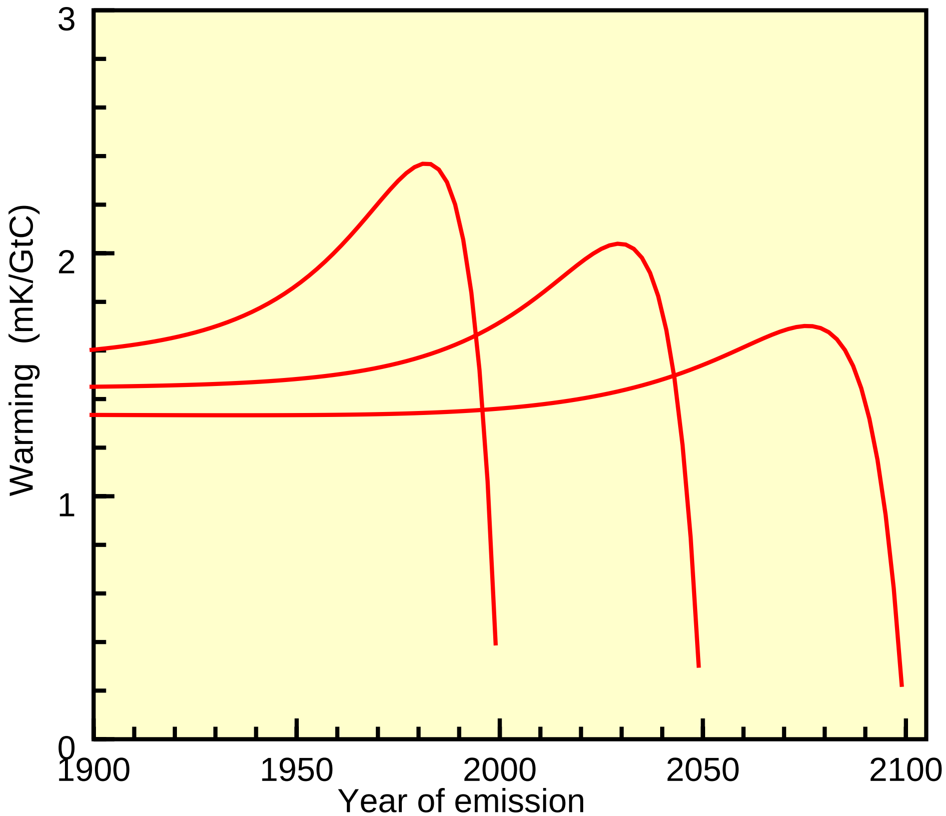
$$\frac{d}{dt}y_m = \frac{\partial}{\partial a}g_m(\{x_k\}, a, t) + \sum_n \frac{\partial}{\partial x_n}g_m(\{x_k\}, a, t)\, y_n$$

$$\frac{d}{dt}y_{m,p} = \frac{\partial}{\partial a_p}g_m(\{x_k\}, \underline{a}, t) + \sum_n \frac{\partial}{\partial x_n}g_m(\{x_k\}, \underline{a}, t)\, y_{n,p}$$

Analysing the Brazilian Proposal

- Construct simple climate model
- Construct linearisation (e.g. by automatic differentiation)
- Calculate sensitivities, either by brute force application of linearised model or by explicit adjoint model.
- Apply sensitivities to histories of emissions from each nation
- Repeat for all greenhouse gases

Results: Fréchet Derivatives



Assumes IS92a emissions. Represents temperature by response function. Linear responses for ocean and biotic carbon, coupled non-linearly to atmospheric CO_2 (as in CSIRO study).

$$\frac{\partial}{\partial E(t)} T(\tau) \text{ for } \tau = 2000,\ 2050,\ 2100.$$

Decrease as $t \to \tau$ shows 'committed warming'.
At any time, warming from the most recent releases is yet to happen.

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Implications

- For a given indicator, T^*, calculation of $S(t)$ allows attribution to any nation.

- $S(t)$ most efficiently calculated from adjoint model, but for multiple indicator times, tangent linear model not too inefficient.

- Sensitivity of T_j^* to model uncertainties can be obtained as second derivatives.

- Sensitivity of T_j^* to uncertainties in emissions can be obtained as

$$\mathrm{Var}[T_j^*] = \int\int S(t)\mathrm{Cov}[E_j(t), E_j(t')]S(t')\, dt'\, dt$$

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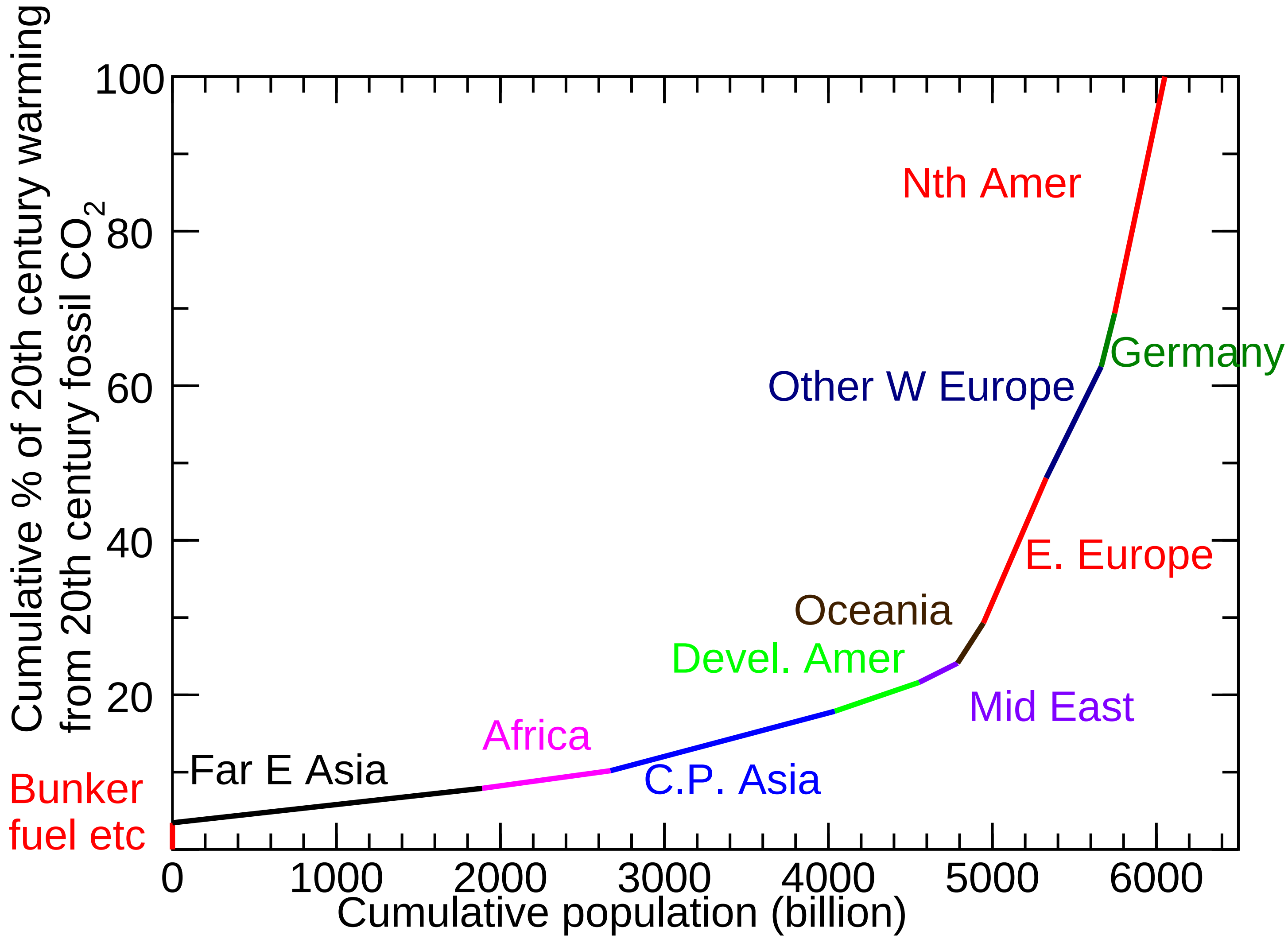
Attribution 2000, fossil CO_2 only



Cumulative responsibility for the fossil CO_2 component of warming vs cumulative population.

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Concluding remarks

- An interesting example of adjoint sensitivity analysis and automatic differentiation
- The Brazilian Proposal is on the agenda, for formal consideration by Conference of Parties (to the Climate Change Convention) in 2008
- Expert working group is extending calculations to include all major greenhouse gases, with detailed national attribution

Further Information

Andreas Griewank, 2000, *Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation*, (SIAM, Philadelphia).

MATCH website (Brazilian Proposal): *http://www/match-info.net*

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