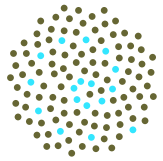


Attribution of Responsibility for Climate Change

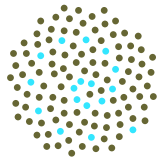
The Mathematics Behind the Brazilian Proposal

Ian G. Enting

MASCOS

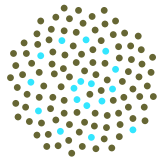
The University of Melbourne

Acknowledgments



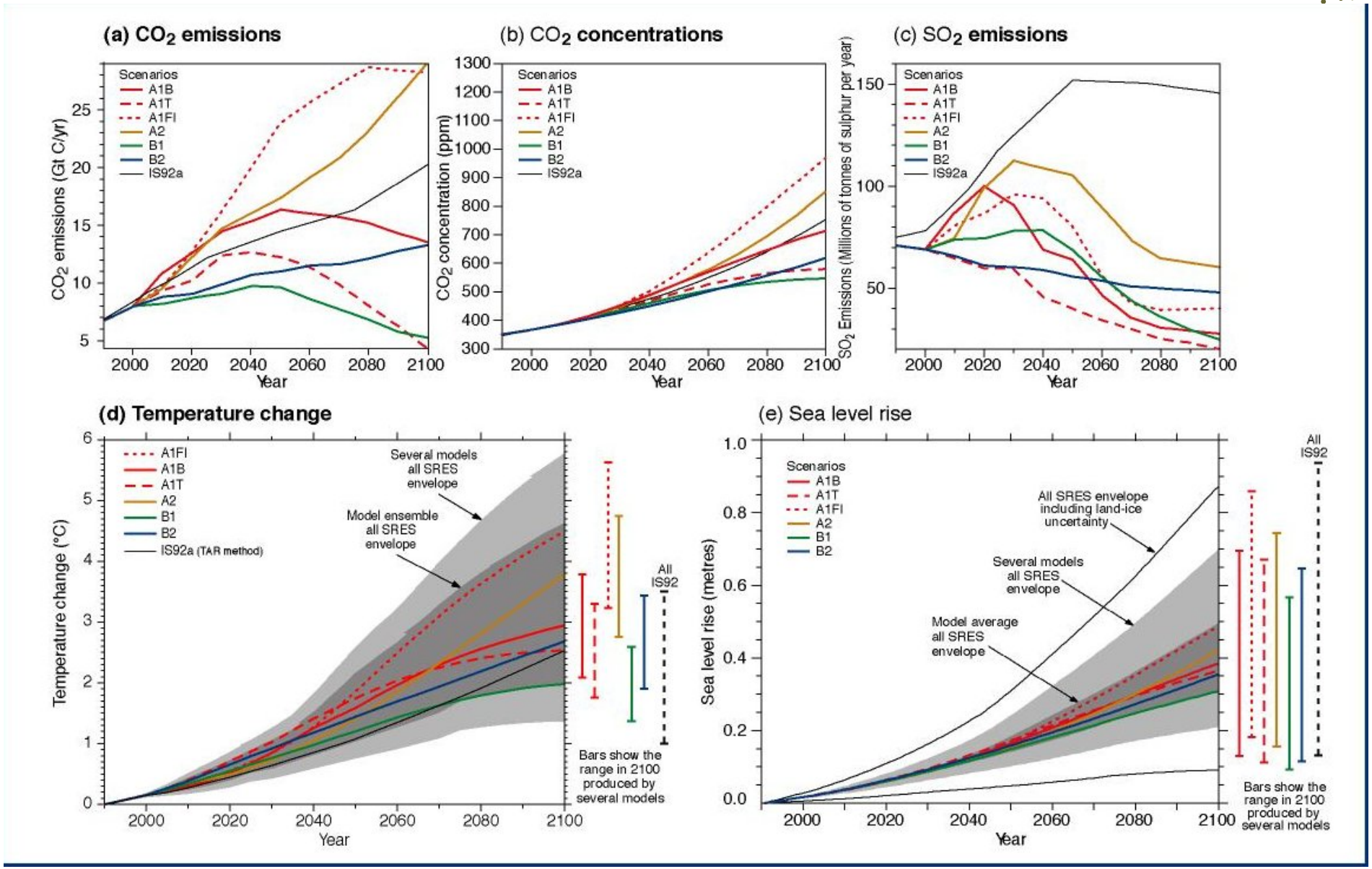
- The Center of Excellence for Mathematics and Statistics of Complex Systems (MASCOS) is funded by the Australian Research Council (ARC).
- My fellowship at MASCOS is supported by CSIRO through a sponsorship agreement.
- Collaborators: Cathy Trudinger of CSIRO Marine and Atmospheric Research and members of the MATCH working group on the Brazilian Proposal.

Summary



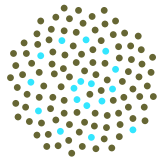
- Time-scales for the greenhouse effect
  - committed warming
- The Brazilian Proposal
  - setting reduction targets in proportion to responsibility
- Adjoint modelling & automatic differentiation
  - efficient calculation of sensitivities
  - operator overloading
- Analysing the Brazilian proposal
  - Who's to blame for the greenhouse effect?

The causal chain



From IPCC Third Assessment report

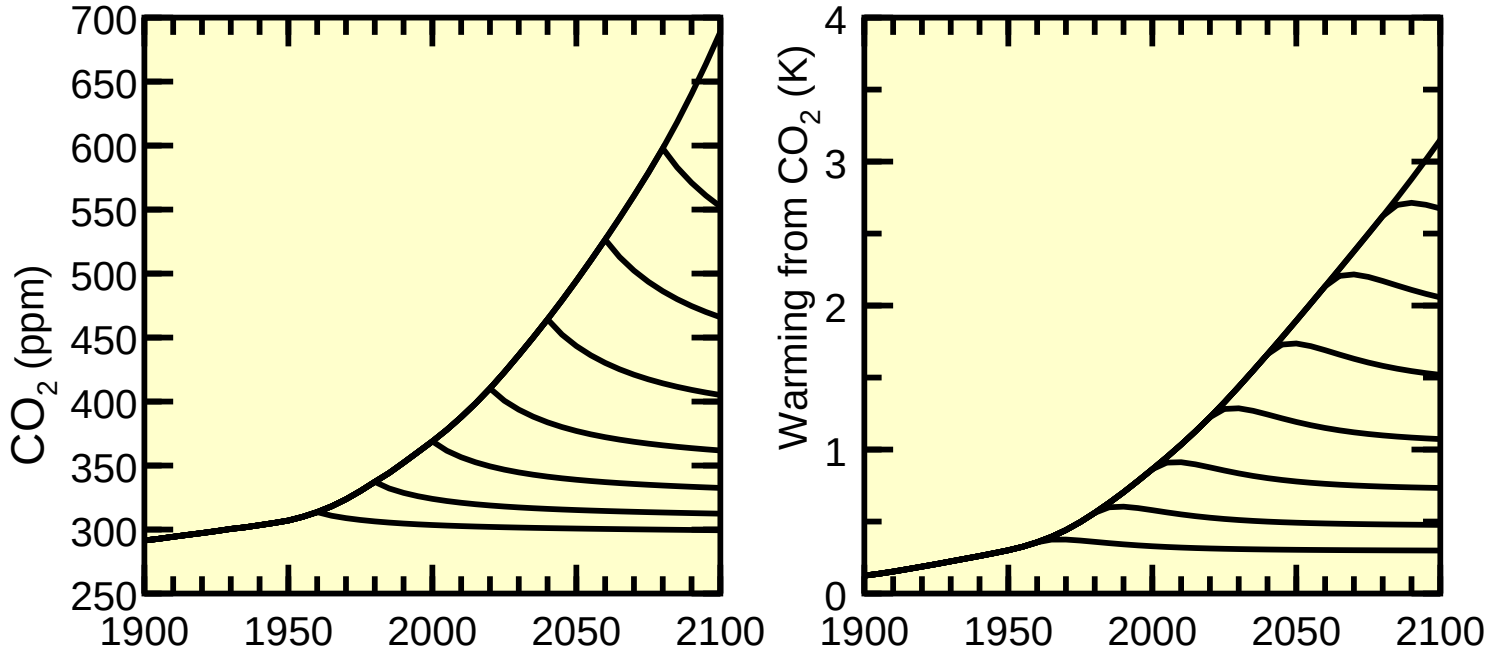
IPCC is Cautious



- The IPCC requirement for a consensus evaluation of 'well-established science' can mean that IPCC reports lag behind the forefront of science, sometimes by more than a decade.
- Similarly, Al Gore's book and film are very careful to avoid premature speculation about extreme possibilities.
- Recent study (Science Express, 1/2/07) indicates that changes since 2001 are tracking the high end of IPCC projections.
- Overview by Barrie Pittock on mechanisms that may imply current under-estimation of change.
- For 'alarmist' views, see 'Pentagon scenarios'.

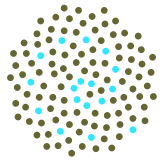
Timescales

CO_2 concentrations and consequent warming, partitioned according to time of emission.



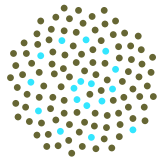
Lowest bands are from pre-1960 emissions, next from 1960 to 1980 emissions, etc.
Increase in contribution to warming after time of emissions from 'committed warming' effect.

Some key dates



- circa 1960: Keeling measures CO_2 growth
- Oct 1985. *it is now believed that in the first half of the next century a rise of global mean temperature could occur which is greater than any in man's history*. (UNEP/WMO/ICSU conference, Villach).
- 1998. IPCC established by WMO and UNEP
- UNFCCC. Rio 1992. In force 21/3/99
- 1995. CoP-1, Berlin Mandate (to negotiate a protocol)
- Kyoto Protocol: 11/12/97. came into force 16/2/05.
- 2001 (IPCC) balance of evidence suggests discernable human influence
- Feb 2007 (IPCC): unequivocal attribution to human agency of most of the warming since 1950.
- 2008-2012. Kyoto Protocol: first commitment period

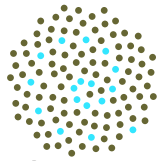
Mitigation frameworks



- Kyoto, specified percentage reductions
- Contraction and convergence
- Sector-based targets – e.g. the Tryptique framework underlying target-setting within the 'EU bubble'

Note that measures such as carbon trading, clean development mechanism or the McKibbin-Wilcoxen multi-level permits are not primarily about mitigation levels, they are about economically-efficient implementation.

Brazilian Proposal



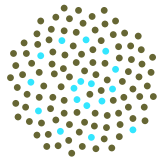
Tabled by Brazil during negotiations leading to Kyoto Protocol — Flicked-passed to Subsidiary Body for Scientific and Technical Advice (SBSTA).

Proposes that emission reduction targets should be proportional to nations' relative responsibility for the greenhouse effect.

Issues:

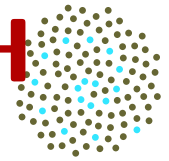
- Indicator? What quantity is used as a measure of the greenhouse effect?
- For what period of emissions is responsibility attributed?
- How are non-linear responses attributed?

Political significance



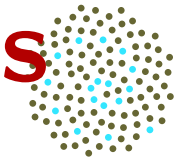
- It captures the historical responsibility of developed nations (for global warming) and the requirement of developed nations to take the lead in combating climate change — i.e. as prescribed by the UN Framework Convention on Climate Change.
- It provides a formula that can apply to all nations
- Therefore it provides a way of engaging developing nations as they develop.

Expert Working Group: MATCH



- Initially under the auspices of SBSTA, initial meeting in Brazil
- Now an informal working group with government support from UK, FRG, Norway. Expert Meetings: Bonn, 2001; Bracknell, UK, 2002; Berlin, 2003; Cologne, 2004; Rio de Janeiro, 2005; Reading, UK 2005; Louvain-la-Neuve, Belgium 2006; Cologne, 2006.
- Joint paper published:
  den Elzen et al. Analysing countries contributions to climate change: scientific and policy-related choices. *Environmental Science and Policy* 8: 614–635 (2005).
- Initial results presented to SBSTA, May 2006. Final results scheduled for presentation to SBSTA late 2007.

Brazilian Proposal as Derivatives



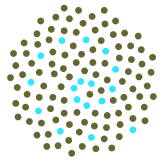
As example, use indicator $T^* = T_{CO2}(2000)=$ warming in 2000 from CO_2 emissions.
T^* is to be attributed to emissions $E_j(t)$ from country j with $E(t) = \sum_j E_j(t)$.

Differential attribution to country j of emissions at time t is

$$\frac{\partial T^*}{\partial E_j(t)} E_j(t) = \frac{\partial T^*}{\partial E(t)} E_j(t) = S(t) E_j(t)$$

where $S(t)$ is a Fréchet derivative.
Cumulated attribution: $T_j^* = \int S(t) E_j(t)\, dt$

Aims of adjoint modelling

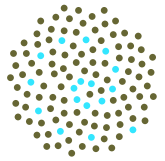


Aim is to simplify calculations by separating parametric differentiation from model integration, expressed here in terms of Green's function $\mathcal{G}$ of $\mathcal{L}\underline{u}(.) = \underline{f}(.)[\underline{a}]$.

Considers $\nabla_{\underline{a}}\langle \underline{w}(.)|\underline{u}(.)[\underline{a}]\rangle$
where $\underline{u}(.)[\underline{a}] = \mathcal{G}\underline{f}(.)[\underline{a}]$ (or $\mathcal{L}\underline{u}(.) = \underline{f}(.)[\underline{a}]$)

Transforms as
$\nabla_{\underline{a}}\langle \underline{w}(.)|\underline{u}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle \underline{w}(.)|\mathcal{G}\underline{f}(.)[\underline{a}]\rangle =$
$\nabla_{\underline{a}}\langle \mathcal{G}^{\dagger}\underline{w}(.)|\underline{f}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle \underline{v}(.)|\underline{f}(.)[\underline{a}]\rangle$
where $\underline{v}(.) = \mathcal{G}^{\dagger}\underline{w}(.)$ defines a single function $\underline{v}(.)$ with no dependence on $\underline{a}$.

Principles of adjoint modelling



Given $\underline{u}(.)[\underline{a}] = \mathcal{G}\underline{f}(.)[\underline{a}]$, where typically $\mathcal{L}\underline{u}(.)$ is linearisation of a more general model:

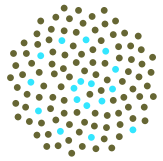
Formally:

$\nabla_{\underline{a}}\langle\underline{w}(.)|\underline{u}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle\underline{w}(.)|\mathcal{G}\underline{f}(.)[\underline{a}]\rangle =$
$\nabla_{\underline{a}}\langle\mathcal{G}^{\dagger}\underline{w}(.)|\underline{f}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle\underline{v}(.)|\underline{f}(.)[\underline{a}]\rangle$
with $\underline{v}(.) = \mathcal{G}^{\dagger}\underline{w}(.)$

In practice, used as $\mathcal{L}\underline{u}(.) = \underline{f}(.)[\underline{a}])$

$\nabla_{\underline{a}}\langle\underline{w}(.)|\underline{u}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle\mathcal{L}^{\dagger}\underline{v}(.)|\underline{u}(.)[\underline{a}]\rangle =$
$\nabla_{\underline{a}}\langle\underline{v}(.)|\mathcal{L}\underline{u}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle\underline{v}(.)|\underline{f}(.)[\underline{a}]\rangle$
with $\underline{w}(.) = \mathcal{L}^{\dagger}\underline{v}(.)$ giving equations for adjoint model.

Applying adjoint modelling



Differentiation (only case used in this talk)

$$\nabla_{\underline{a}}\langle \underline{w}(.)|\mathcal{G}\underline{f}(.)[\underline{a}]\rangle = \nabla_{\underline{a}}\langle \mathcal{G}^{\dagger}\underline{w}(.)|\underline{f}(.)[\underline{a}]\rangle$$

Gradients for soft constraints .

$$\nabla_{\underline{a}}\langle \underline{Hu} - \underline{z}|\underline{Hu} - \underline{z}\rangle = 2\nabla_{\underline{a}}\langle \underline{Hu}_0 - \underline{z}|\underline{Hu}\rangle =$$
$$2\nabla_{\underline{a}}\langle \underline{Hu}_0 - \underline{z}|\underline{H}\mathcal{L}\underline{f}\rangle = 2\nabla_{\underline{a}}\langle (\underline{H}\mathcal{L})^{\dagger}(\underline{Hu}_0 - \underline{z})|\underline{f}\rangle$$

Gradients, with hard constraints: $\mathcal{L}\underline{u}(.) = 0$

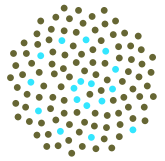
$$\Theta^* = \Theta - \langle \underline{v}(.)|\mathcal{L}\underline{u}(.)\rangle$$

The function $\underline{v}(.)$ is the Lagrange multiplier.

$\nabla_{\underline{u}}\Theta^* = \nabla_{\underline{u}}\Theta - \nabla_{\underline{u}}\langle \mathcal{L}^{\dagger}|\underline{u}(.)\rangle$, whence

$\mathcal{L}^{\dagger}\underline{v}(.) = \nabla_{\underline{u}}\Theta$ — the adjoint equations define the Lagrange multiplier

Tangent Linear Model (TLM)



For N DEs: $\frac{d}{dt}x_j = g_j(\{x_k\}, a, t)$ for $j = 1, N$

we can define sensitivities as

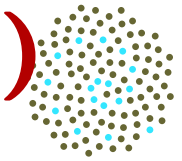
$y_j = \frac{\partial}{\partial a}x_j$ for $j = 1, N$ or $y_{j,p} = \frac{\partial}{\partial a_p}x_j$

to give 'tangent linear model(s)':

$$\frac{d}{dt}y_m = \frac{\partial}{\partial a}g_m(\{x_k\}, a, t) + \sum_n \frac{\partial}{\partial x_n}g_m(\{x_k\}, a, t)\, y_n$$

$$\frac{d}{dt}y_{m,p} = \frac{\partial}{\partial a_p}g_m(\{x_k\}, \underline{a}, t) + \sum_n \frac{\partial}{\partial x_n}g_m(\{x_k\}, \underline{a}, t)\, y_{n,p}$$

Algorithmic Differentiation (AD)



Differentiation by successive use of chain rule.
For binary operation $c = f(a, b)$,

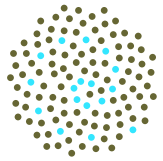
$$\frac{\partial c}{\partial \alpha} = \frac{\partial f}{\partial a} * \frac{\partial a}{\partial \alpha} + \frac{\partial f}{\partial b} * \frac{\partial b}{\partial \alpha}$$

$$\text{e.g.} \quad c = a + b \quad \rightarrow \frac{\partial c}{\partial \alpha} = \frac{\partial a}{\partial \alpha} + \frac{\partial b}{\partial \alpha}$$

$$\text{or} \quad c = a * b \quad \rightarrow \frac{\partial c}{\partial \alpha} = b * \frac{\partial a}{\partial \alpha} + a * \frac{\partial b}{\partial \alpha}$$

Automatically derives TLM: converting program to code for derivatives, one operation at a time.

Approaches to AD



Hand-code program to calculate derivatives — laborious, error-prone and must be repeated each time the model changes.

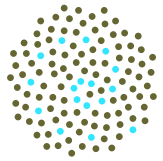
Symbolic algebra (e.g. Mathematica) — problematic for adjoints.

Tangent/adjoint compilers — transform source into code for tangent or adjoint models.

Operator overloading to produce a 'script' that is analysed to give code for the derivatives.

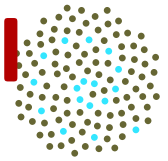
Use operator overloading capabilities directly — straightforward for tangent-linear-model, but restricted applicability to adjoint models.

AD by operator overloading



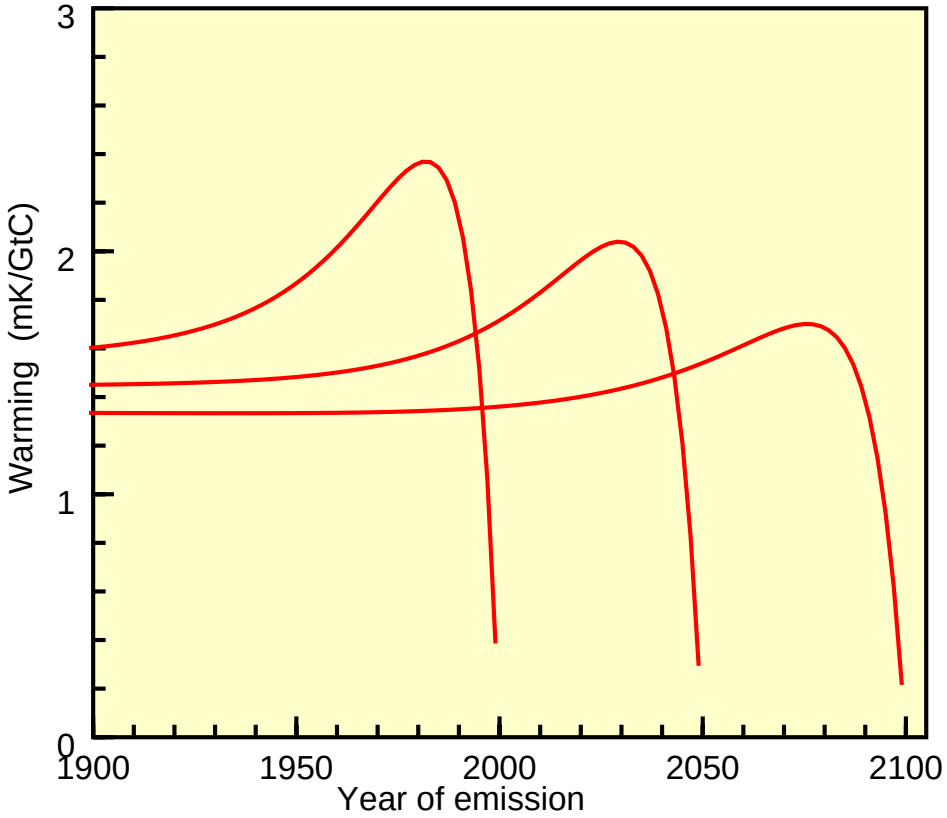
- Needs object-oriented language, C++ or F90
- Define new types that include derivative information
- Use overloading to define operations on these types
- Modify type declarations in original model code to invoke new types
- Set up requisite I/O for derivative information

Analysing the Brazilian Proposal



- Construct simple climate model
- Construct linearisation (e.g. by automatic differentiation)
- Calculate sensitivities, either by brute force application of linearised model or by explicit adjoint model.
- Apply sensitivities to histories of emissions from each nation
- Repeat for all greenhouse gases

Results: Fréchet Derivatives



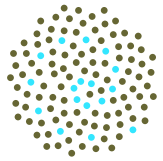
Assumes IS92a emissions.

Represents temperature by response function. Linear responses for ocean and biotic carbon, coupled non-linearly to atmospheric CO_2 (as in CSIRO study).

$\frac{\partial}{\partial E(t)} T(\tau)$ for $\tau = 2000, 2050, 2100$.

Decrease as $t \to \tau$ shows 'committed warming'.
At any time, warming from the most recent releases is yet to happen.

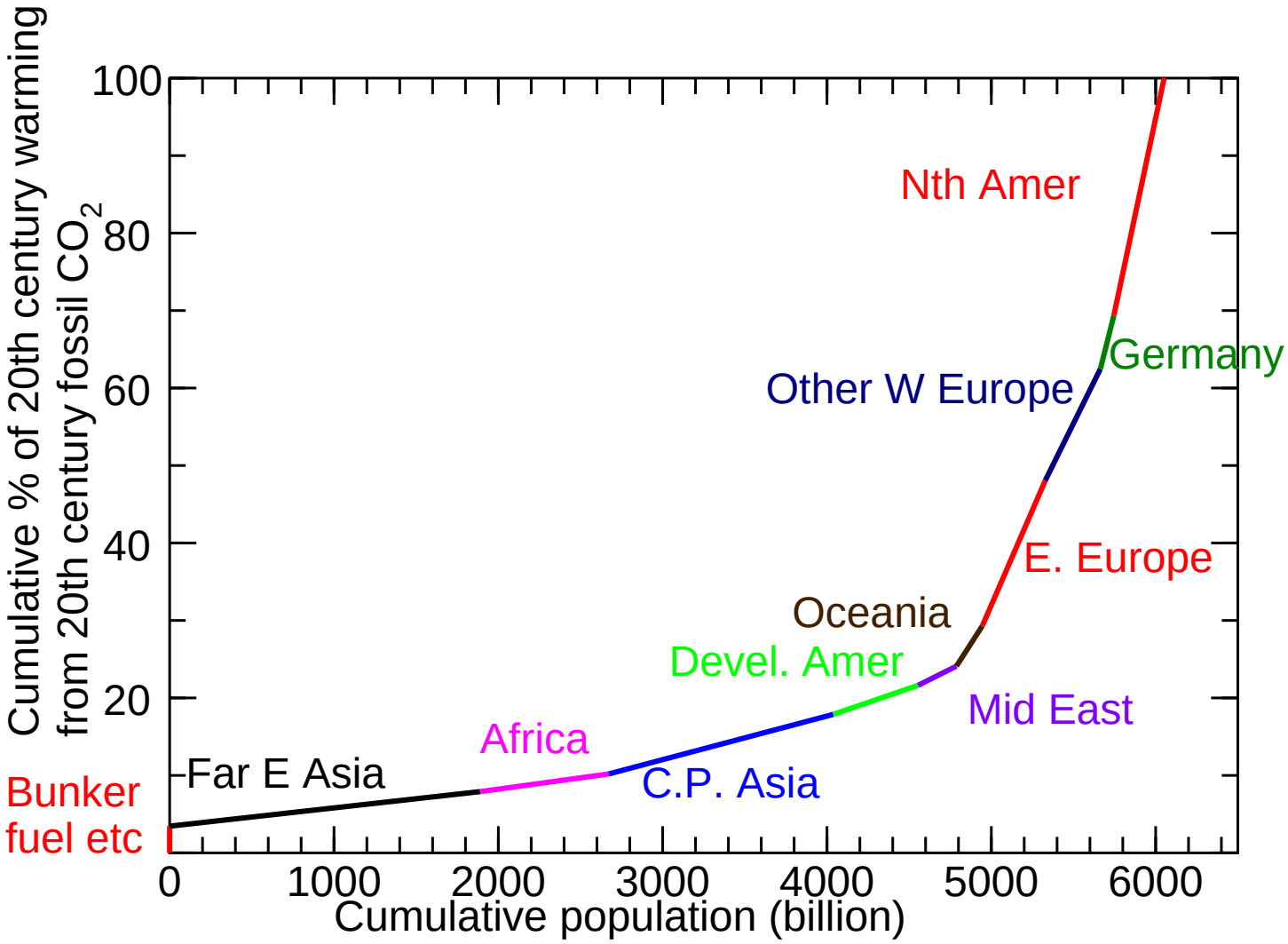
Implications



- For a given indicator, T^*, calculation of $S(t)$ allows attribution to any nation.
- $S(t)$ most efficiently calculated from adjoint model, but for multiple indicator times, tangent linear model not too inefficient.
- Sensitivity of T_j^* to model uncertainties can be obtained as second derivatives.
- Sensitivity of T_j^* to uncertainties in emissions can be obtained as

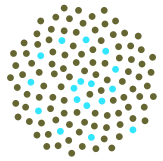
$$\mathrm{Var}[T_j^*] = \int\int S(t)\mathrm{Cov}[E_j(t), E_j(t')]S(t')\,dt'\,dt$$

Attribution 2000, fossil CO_2 only



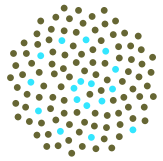
Cumulative responsibility for the fossil CO_2 component of warming vs cumulative population.

Concluding remarks



- An interesting example of adjoint sensitivity analysis and automatic differentiation
- The Brazilian Proposal is on the agenda, for formal consideration by Conference of Parties (to the Climate Change Convention) in 2008
- Expert working group is extending calculations to include all major greenhouse gases, with detailed national attribution

Further Information



MATCH website (Brazilian Proposal):

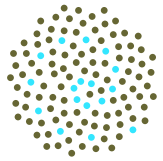
http://www/match-info.net

Andreas Griewank, 2000, *Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation*, (SIAM, Philadelphia).

I.G. Enting, 2005, *Automatic differentiation in the analysis of strategies for mitigation of global change*, International Congress on Modelling and Simulation, Melbourne, 2005. Ed. A. Zerger and R. M. Argent, 7pp
http://www.mssanz.org.au/modsim05/papers/enting.pdf

My AD talk from Berkeley is on my website.

Class Definitions

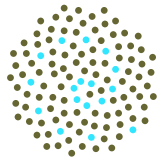


Fragment of C++ class definition to implement operator overloading:

```
class Xvar{
public :
static const int ns = _NUMDERIVS+1;
double xs[_NUMDERIVS+1];
Xvar operator*(Xvar);
...
};

Xvar Xvar::operator*(Xvar b){ Xvar c;
for (int i=1; i < ns; i++)
    c.xs[i] = xs[i]*b.xs[0]+xs[0]*b.xs[i];
c.xs[0] = xs[0]*b.xs[0];
return c;} ;
...
```

Usage



Original

```
double F_co2(double c){
double a;
a = log(c/280.0)*5.35;
return a;
};
...
double cc;
...
ff = F_CO2(cc)
```

Transformed

```
Xvar F_co2(Xvar c){
Xvar a;
a = log(c/280.0)*5.35;
return a;
};
...
Xvar cc;
// Derivatives wrt
// initial value of cc
cc.set(280,1);
...
ff = F_CO2(cc)
```