# Carbon-climate feedbacks: Measurements Mathematics Modelling

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# Summary

- Mathematics and Modelling
  - Inverse problems
- Mathematics of carbon-climate feedbacks
- Modelling approaches.

# Mathematics and modelling

**Real-world system → Math. model → Computer model**

Relation between real world and mathematical and computer models. Distinction is useful because:

**1** Emphasises two different types of testing: *validation*: ensuring that the mathematical model represents the real world and *verification* ensuring that computer model implements mathematical model.

**2** Possible, and often desirable, to manipulate the mathematical representation during modelling.

**Real-world → Mathematical model**
**↓**
**Approximate math. model → Computer model**

# Towards model analysis systems

Transformation of the mathematical model often leads to multiple computer models:

| | | | | |
|---|---|---|---|---|
| Real-world system | → | Mathematical model 1 | → | Computer model 1 |
| | | ↓ | | |
| | | Mathematical model 2 | → | Computer model 2 |
| | | ↓ | | |
| | | Mathematical model 3 | → | Computer model 3 |

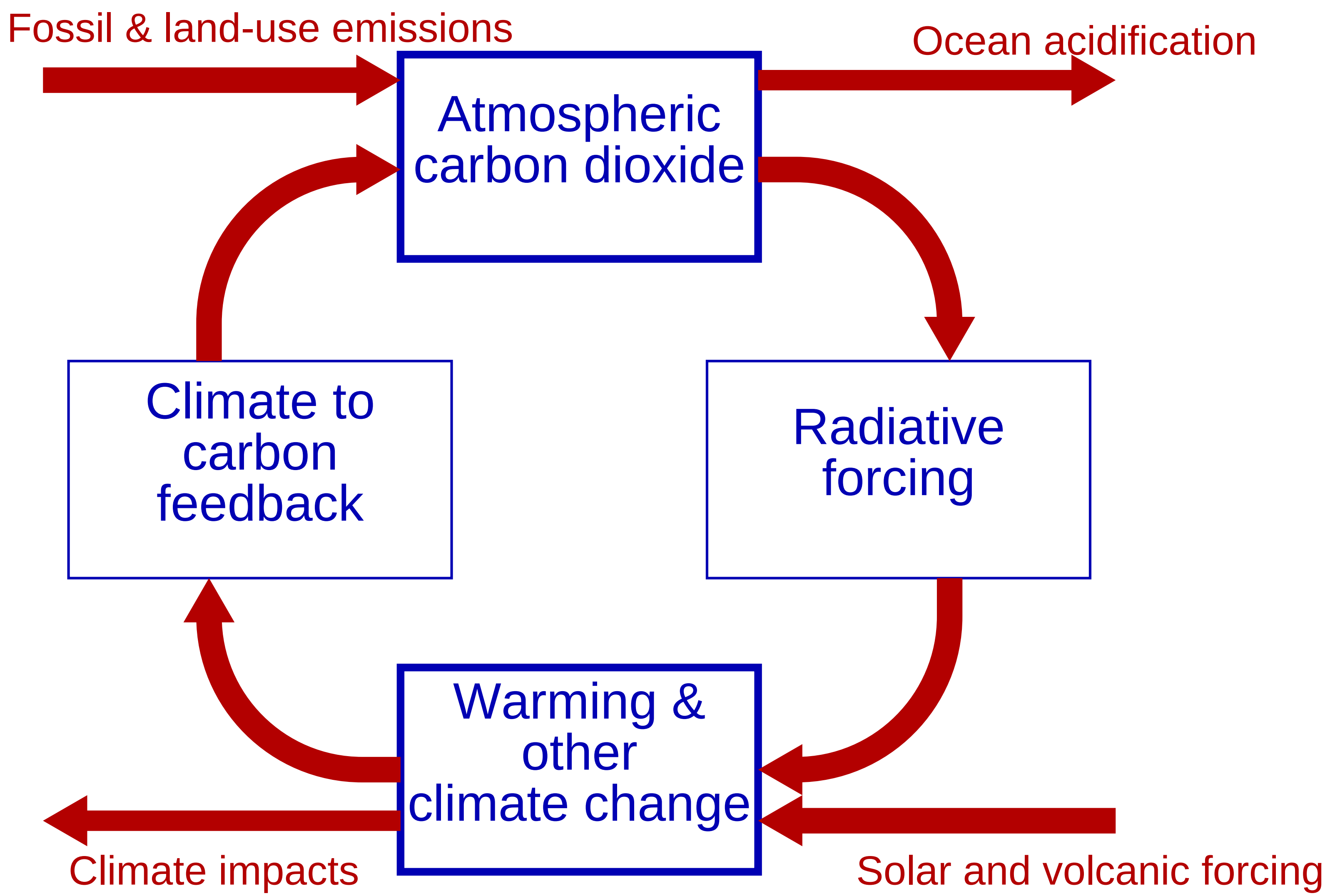
The aim is to achieve something more like:

| | | | | |
|---|---|---|---|---|
| | | Mathematical model 2 | | |
| | | ↑ | ↘ | |
| Real-world system | → | Mathematical model 1 | → | Computer model |
| | | ↓ | ↗ | |
| | | Mathematical model 3 | | |

# IPCC caveats

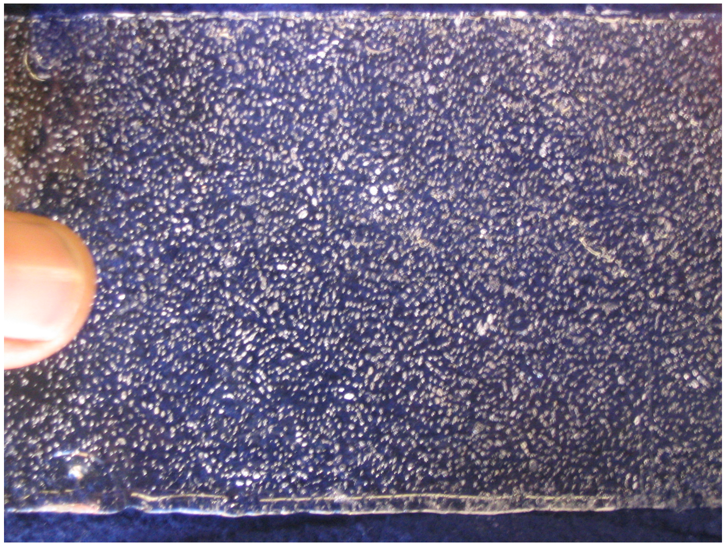
- The magnitude of the positive feedback between climate change and the carbon cycle is uncertain. (AR4: TS.5.5).
- Dynamical processes not included in current models but suggested by recent observations could increase the vulnerability of the ice sheets to warming, increasing future sea level rise. (AR4: TS.5.5).

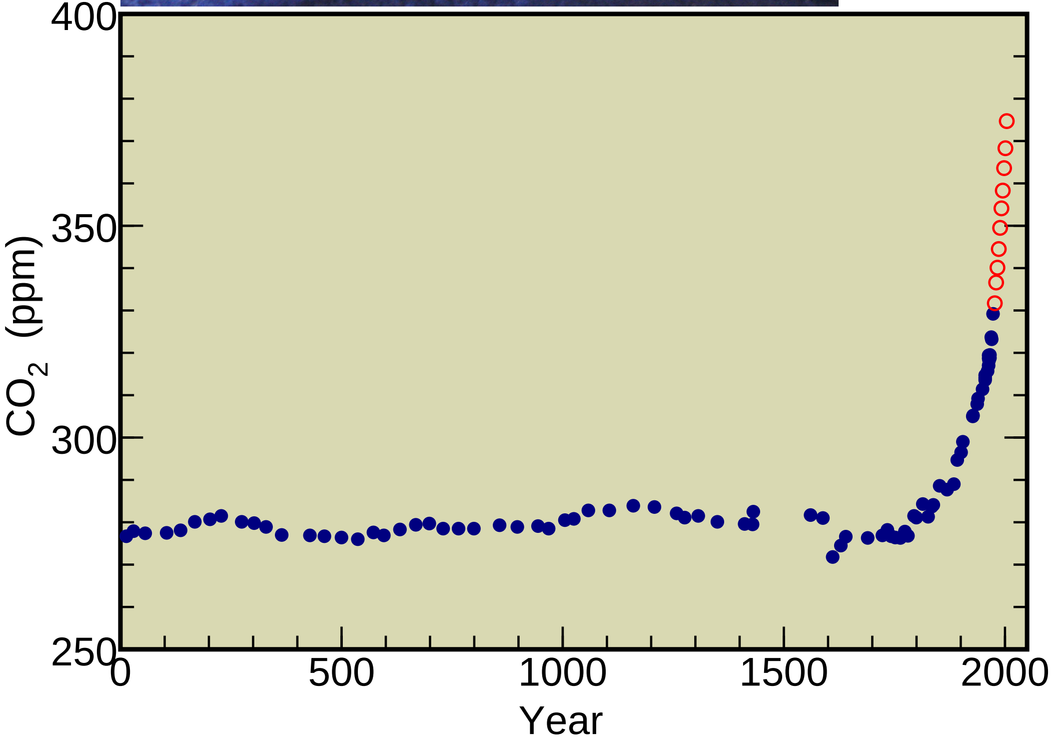
# Feedbacks




From *Twisted: The Distorted Mathematics of Greenhouse Denial.* I Enting, 2007.

# Observing feedbacks



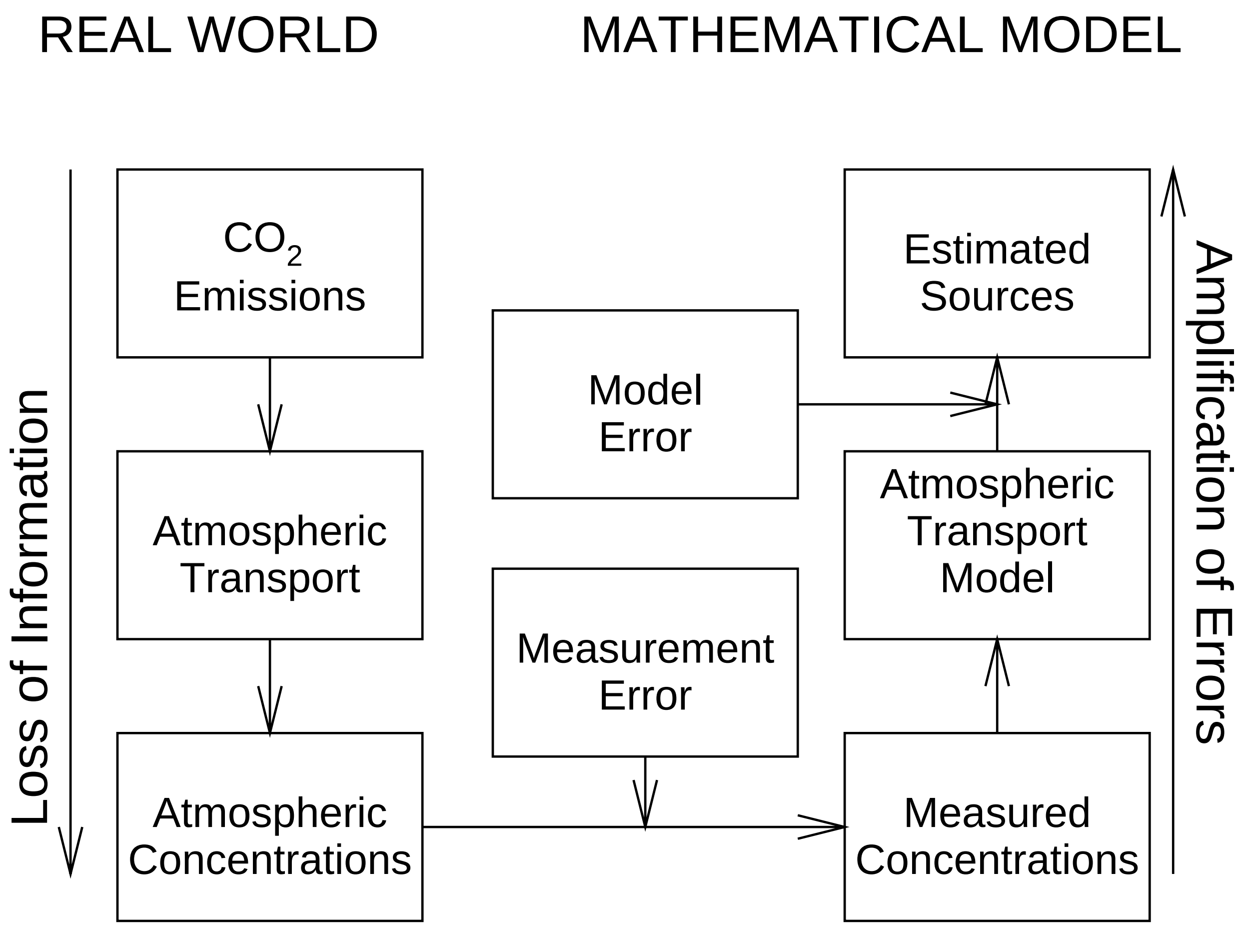



$CO_2$ concentrations from air in bubbles in polar ice and direct atmospheric measurements.

Dip from 1600 to 1800 is climate to carbon feedback from little ice age.

Carbon-13 data show most change was on land, not in oceans. (Trudinger, PhD).

# Inverse problems: the problem




From *Inverse Problems in Atmospheric Constituent Transport.* I Enting, (CUP) 2002.

# Linear analysis of carbon

Linear response function, $R$, defines how concentrations, $C$, respond to source, $S$.

$$Q(t) = C(t) - C(t_0) = \int_{t_0}^{t} R(t - t')\, S(t')\, dt'$$

Forward modelling: calculate $C$ given model response $R$ and sources, $S$.

Two inverse problems:

Deduce $R(t)$ from $C(t)$ and $S(t)$.

Model calibration

Deduce $S(t)$ from $R(t)$ and $C(t)$.

Deconvolution

# Laplace transform

Laplace transforms have similar properties to Fourier transforms in analysing linear systems.

- Transforms from time, $t$ to inverse time variable, $p$
- Convolution relations transform to products
- Integration multiplies transform by $1/p$

Laplace transforms are more appropriate for one-sided causal relationships, than Fourier transforms.

Use lower case to denote Laplace transforms, thus

$$f(p) = \int_0^\infty F(t)\, e^{-pt}\, dt$$

Carbon relations are $q(p) = r(p)\, s(p)$ whence:
$r(p) = q(p)/s(p)$ and $s(p) = q(p)/r(p)$

# Partitioning of carbon

Mass balance of carbon in
Atmosphere $A$, Biosphere, $B$ and Ocean $O$ gives:

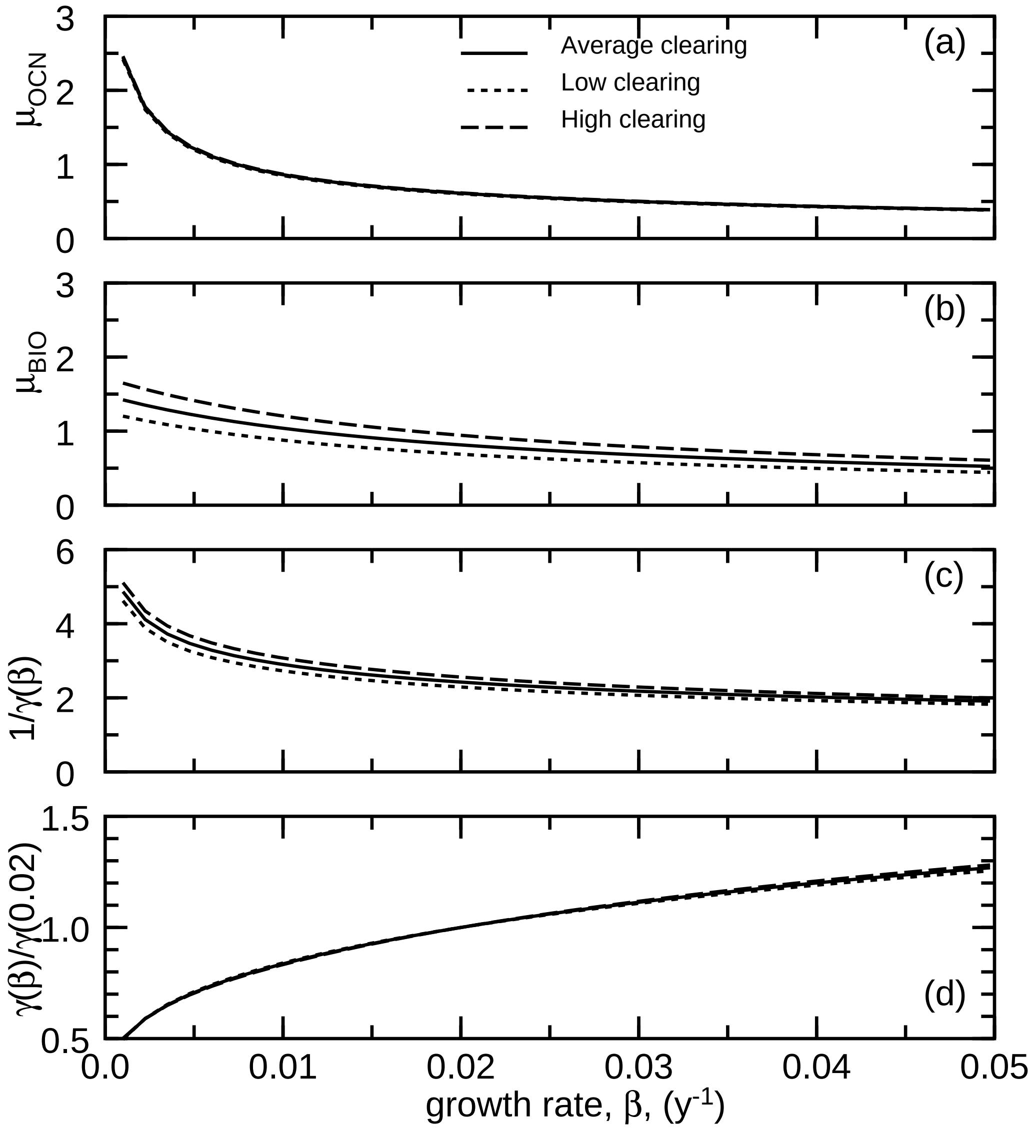
$$\frac{d}{dt}[A + B + O] = \Phi_{\mathsf{Fossil}} + \Phi_{\mathsf{LUCF}} \tag{1}$$

or, using overdot to denote time derivatives, inverse of airborne fraction is

$$\frac{\Phi_{\mathsf{Fossil}} + \Phi_{\mathsf{LUCF}}}{\dot{A}} = 1 + \frac{\dot{B}}{\dot{A}} + \frac{\dot{O}}{\dot{A}}$$

Corresponding relations come from any linear transformation of (1), time-averages, Fourier or Laplace transforms etc, so long as all terms in (1) are transformed in the same way, *including transformations implicit in data, e.g. bubble trapping in ice.*

# Time-dependence of partitioning




Ratios of carbon growth (or release) rates:

(a) ocean: atmosphere

(b) biosphere: atmosphere

(c) fossil:atmosphere

(d) atmosphere:fossil

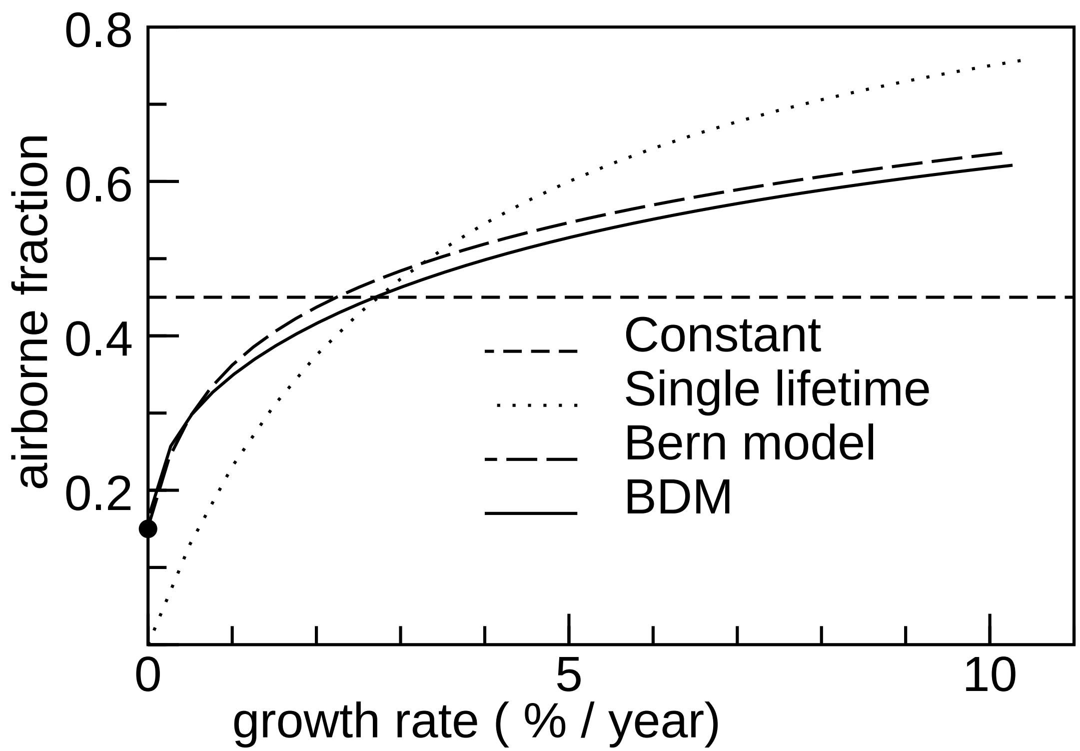
Uncertainties from box model calibrated by C14.

# Information in $CO_2$ data

For exponentially growing emissions:

$$Q(t) = \int_{-\infty}^{t} R(t-t')A\,\mathrm{exp}(\beta t')\,dt'$$
$$= A\,\mathrm{exp}(\beta t)\int_{0}^{t} R(\tau)\,\mathrm{exp}(-\beta\tau)\,d\tau$$

Comparing $CO_2$ emissions and concentrations over 20th century tells you about $r(p \approx 0.02)$ and little more.



# Carbon climate coupling

For a linearised model of the coupled carbon-climate system, warming, $W(t)$, is a response to $CO_2$ perturbation $Q$ and other forcing $F(t)$

$$w(p) = u(p)[f(p) + \alpha q(p)] \qquad (1)$$

A response $H(t)$ describes additional $CO_2$ source from warming:

$$q(p) = r(p)[s(p) + h(p)w(p)] \qquad (2)$$

whence

$$w(p) = \frac{u(p)f(p) + \alpha u(p)r(p)s(p)}{1 - \alpha u(p)r(p)h(p)} \qquad (3)$$

# Quantifying feedback

$$q(p) = \frac{r(p)[s(p) + f(p)h(p)u(p)]}{1 - \alpha u(p)r(p)h(p)}$$

Forcing $f(p)$ or $s(p)$ is amplified by feedback factor: $1/[1 - \alpha u(p)r(p)h(p)]$.

For multi-decadal time-scales, C4MIP gives $1.18 \pm 0.11$ (see AR4,WG1, tbl 7.4, 11 models, range 1.04 to 1.44).

$$\alpha\, u(p)\, r(p)\, h(p) < 1 \qquad \text{for all } p \text{ for stability}$$

$$\alpha\, u(p)\, r(p)\, h(p) = \frac{T_{2*\mathrm{CO}2}}{280 \ln 2} \times \frac{u(p)}{u(0)} [p\, r(p)]\, [h(p)/p]$$

$1 - u(p)/u(0)$ is proportion of committed warming for time-scale $1/p$, $p\, r(p)$ is the $CO_2$ airborne fraction and $h(p)/p$ is feedback response as integrated flux.

# The 20th century

$$q(p) = \frac{r(p)[s(p) + f(p)h(p)u(p)]}{1 - \alpha u(p)r(p)h(p)}$$

Is calibration using $C(t)$ and $S(t)$ giving model characterised by $r(p)$ or $r(p)/[1 - \alpha u(p)r(p)h(p)]$?

- Are models being calibrated with 'incommensurate' data sets (c.f. flux budget vs storage budget issue circa 1990).
- If models reproduce $r(p)/[1 - \alpha u(p)r(p)h(p)]$ then associated feedbacks are not something extra to add when considering 21st century.

# Long term behaviour

(Scheffer, Brovkin and Cox, GRL, 33, L10702, 2006.)

| Scheffer et al | $\frac{\partial T}{\partial C}$ | $\frac{\partial C}{\partial T}$ |
|---|---|---|
| Here | $\frac{T_{2*\mathrm{CO2}}}{280 \ln 2}$ | $[p\, r(p)]\, [h(p)/p]$ as $p \to 0$ |

$T_{2*\mathrm{CO2}} = 3 \pm 1.5$ K implies $\frac{\partial T}{\partial C} = 0.015 \pm 0.007$ K/ppm.
but Scheffer et al neglect $\ln 2$ factor so $\frac{\partial T}{\partial C} = 0.0107$.

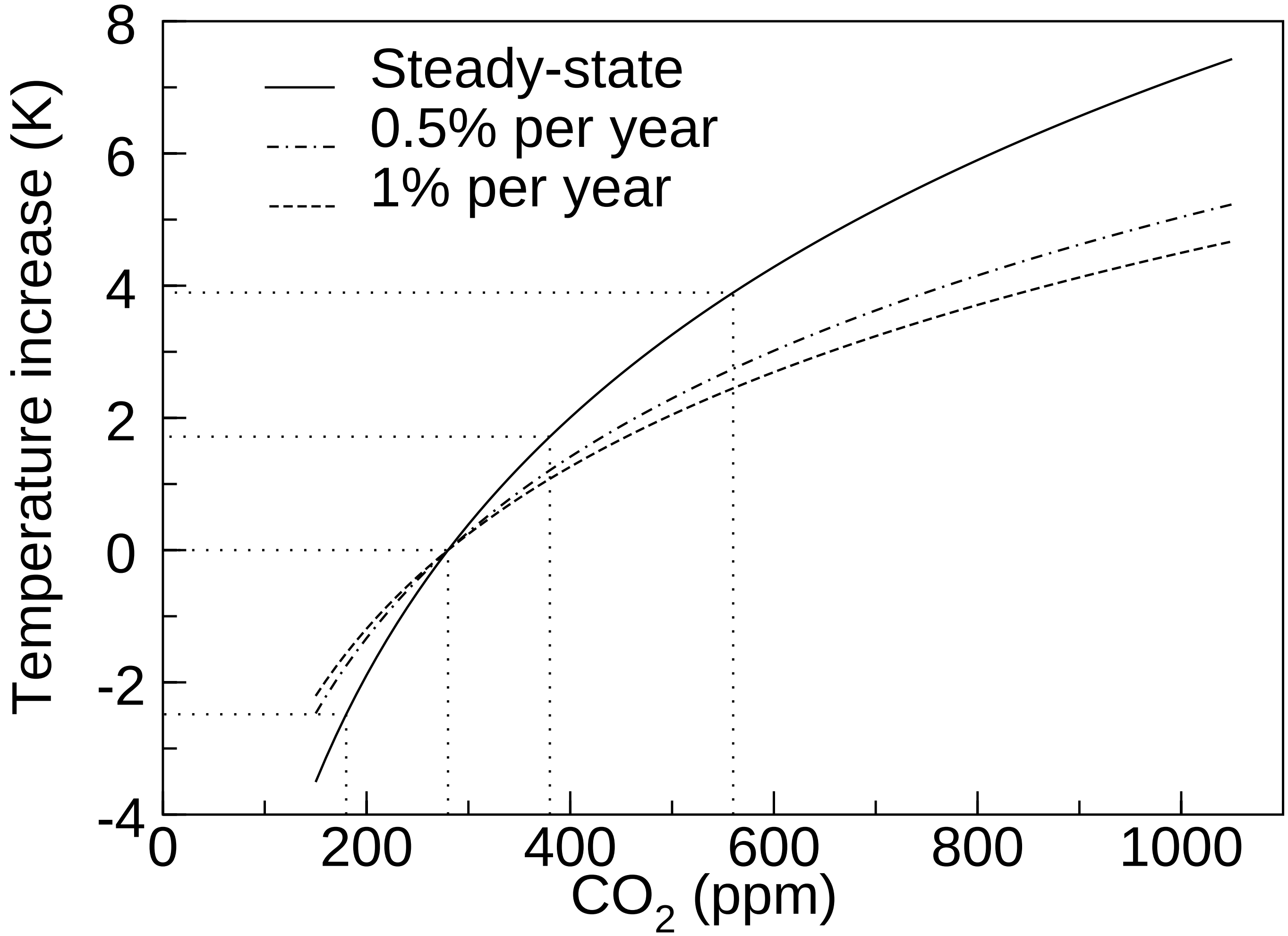
Estimate $\frac{\partial C}{\partial T} \approx \frac{dC}{dt} \Big/ \frac{dT}{dt} \approx \frac{dC}{dt} \Big/ \frac{2}{3}\frac{d}{dt} T_{NH}$

| Period | $\frac{dT}{dt}$ (K/yr) | $\frac{dC}{dt}$ (ppm/yr) | $\frac{\partial C}{\partial T}$ | Feedback |
|---|---|---|---|---|
| 1200–1700 | 0.0003 (NH) | 0.0082 | $\approx 41$ | 1.8 [2.7] |

# Re-consider

- In terms of total change in carbon in Little Ice Age, $\frac{\partial C}{\partial T} \approx 41$ seems too high, and Scheffer et al propose $\frac{\partial C}{\partial T} \approx 12$ on basis of Moberg temperature reconstruction giving amplification of 1.14 (or 1.22 if $\ln 2$ factor included).
- On these scales $u(p)/u(0) \approx 1$

# Quantifying committed warming

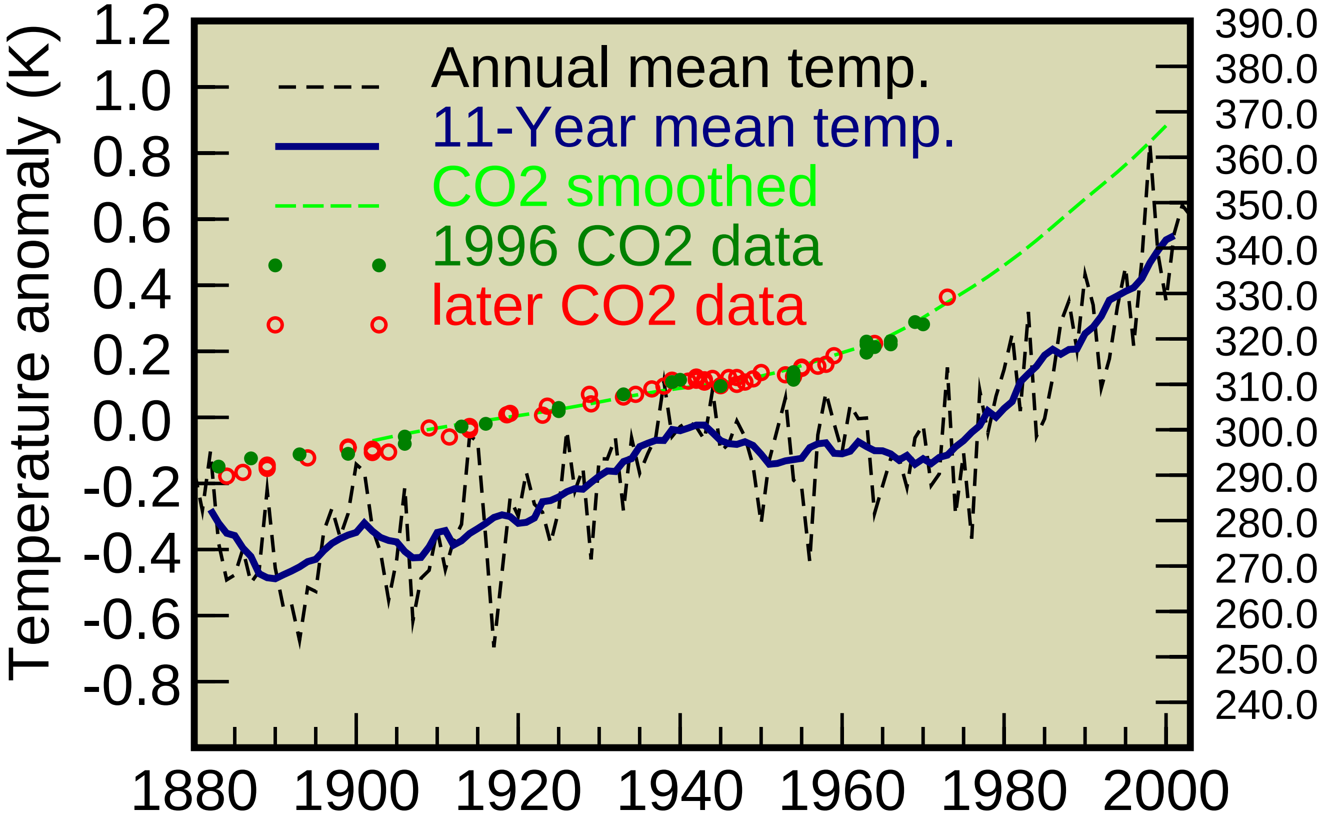



Warming vs $CO_2$ if $CO_2$ increase gives fixed percentage growth in radiative forcing.

# Implications for shorter timescales

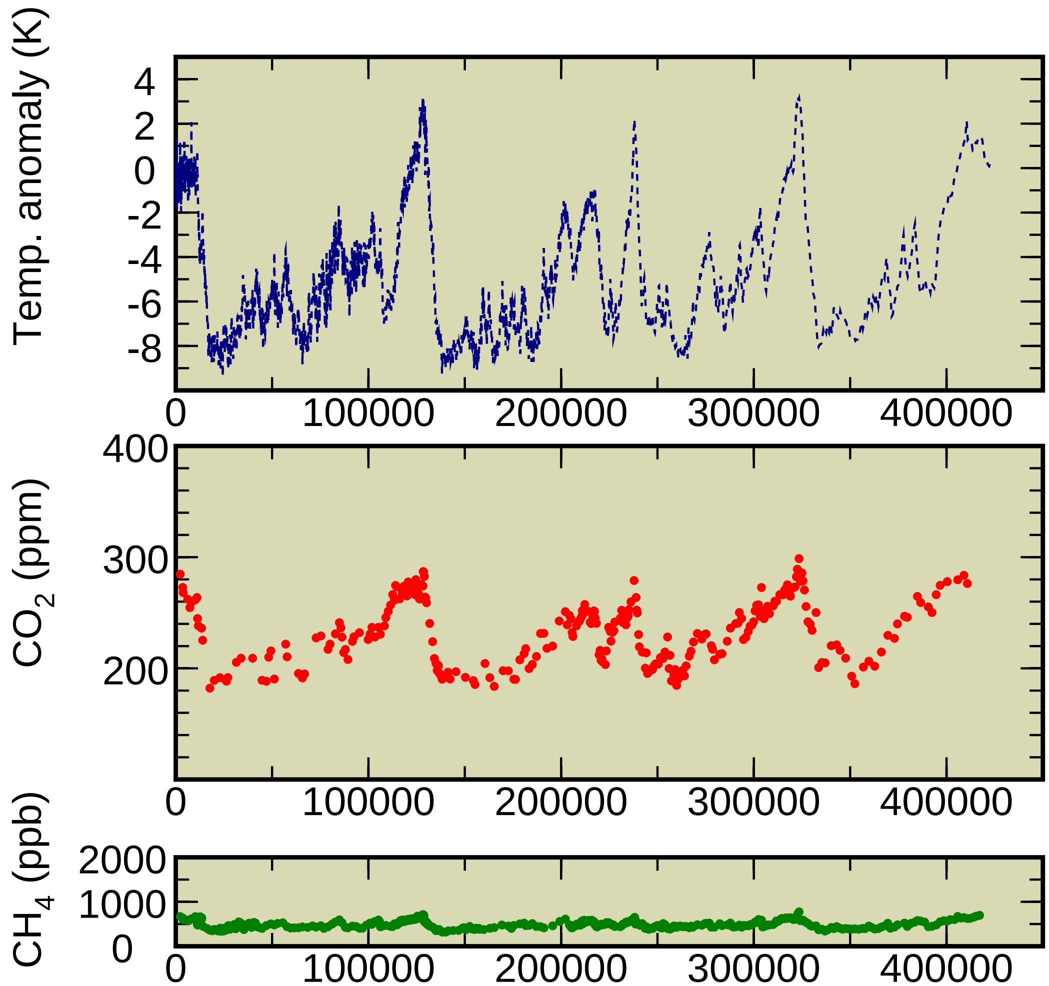
- Committed warming: $u(p)/u(0)$ decreases with increasing $p$
- More importantly, airborne fraction, $p\,r(p)$ increases with increasing $p$.
- Little really known about how $h(p)/p$ (or $h(p)\,r(p)$ should behave, but:
  - Might expect terrestrial processes to equilibrate faster than oceanic.
  - $CO_2$ response to Pinatubo may provide indication for timescales of years.

# Another case?




Law Dome ice core and global temperature data (11-year smoothing of temperature matches smoothing of $CO_2$ in bubble trapping).

# Vostok




Probably limited direct applicability to 21st century processes.

# Modelling strategy

Laplace transform defines large-scale relations and the mathematical structure of the problem.

Quantitative analysis needs to take account of representativeness in time (season, event) and space and role of water balance.

Use process model to identify regions most sensitive to changes on various scales.

# Data for Carbon Cycle Studies

- Air sampling networks interpreted by inverse modelling;
- Satellite data, for quantities such as leaf-area index and phenology
- Terrestrial biosphere models;
- Convective boundary layer measurements;
- Stand-level flux networks;
- Ecosystem experiments;
- Small cuvettes.

From Canadell et al. 2000.

# Key characteristics of statistics

- magnitude;
- degree of correlation between components;
- temporal correlation structure;
- spatial correlation structure;
- distribution;
- mismatches in averaging;
- contribution from model representativeness error.

From Raupach et al. 2005

# Characteristics of terrestrial carbon

- very great spatial heterogeneity
- dominated by local interactions (coupled to atmosphere)
- wide range of time-scales involved

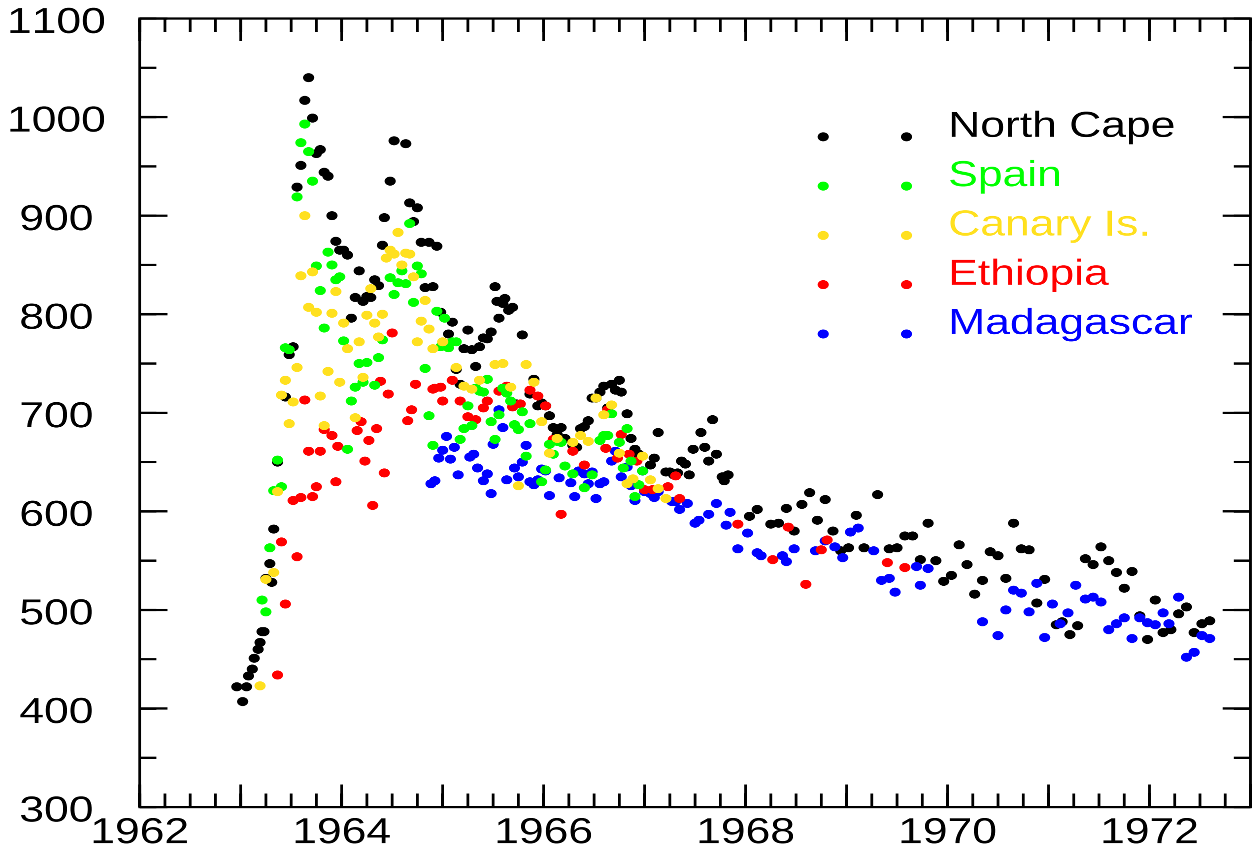
Water in the land-surface has similar characteristics.

# Modelling framework

Successively resolve on

- Global
- Biomes
- Grid
- Tiles within grid
- Co-located types (e.g. fixers vs non-fixers)

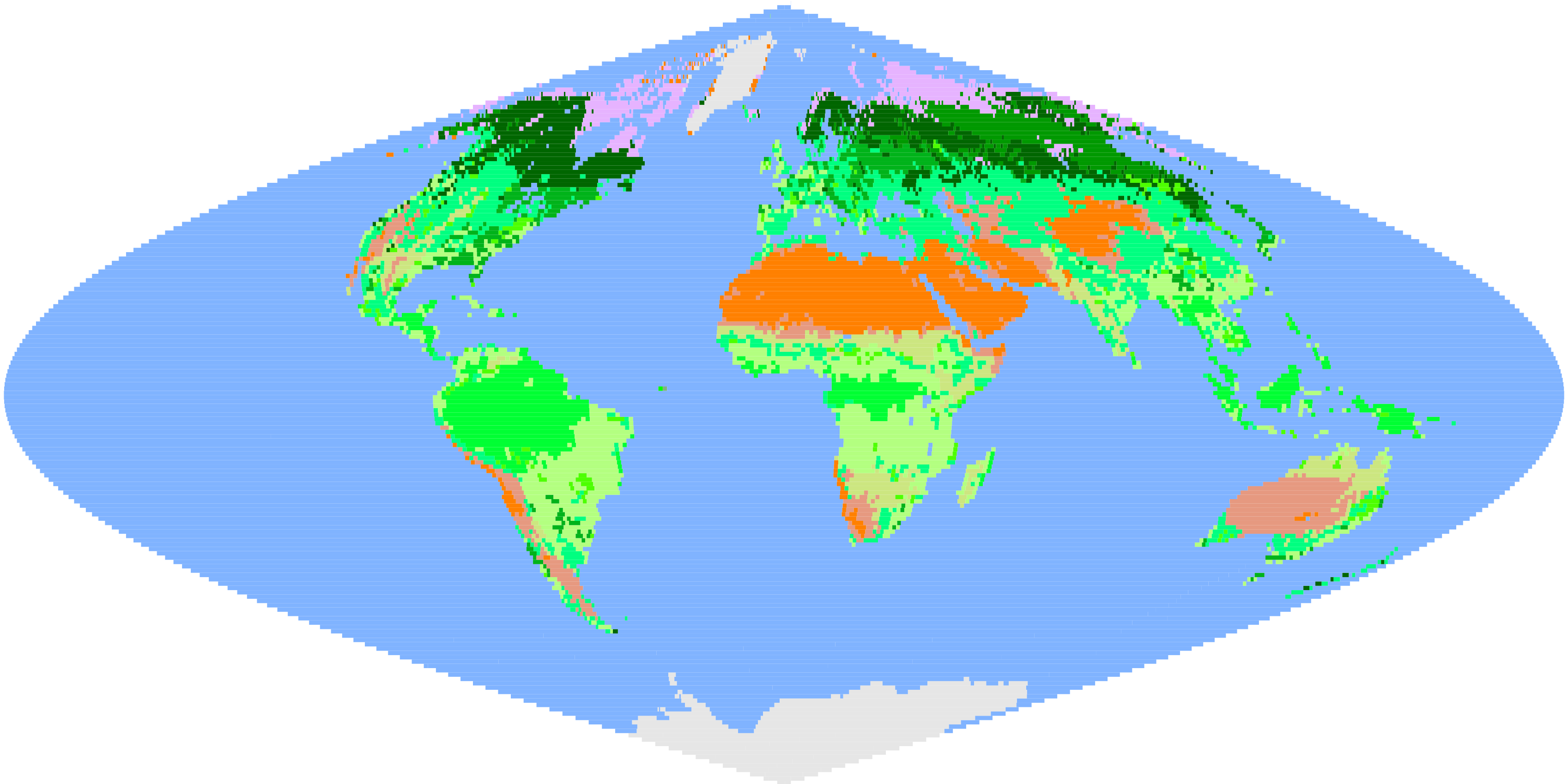
# Bomb-$^{14}$C, with seasonal variation




Surface values, ($\Delta^{14}$C in per mil) from Nydal and Lovseth, JGR, 88C, 3621, 1983. CDIAC NDP057.

In times of isotopic disequilibrium, $^{14}$C data give information about gross terrestrial fluxes. Randerson et al, (2002), analysed these data mainly as a constraint on seasonality of stratosphere-troposphere exchange.

# CASA vegetation types



Seasonal modulation of bomb $^{14}C$ 'spike' gives a low-pass spatial filtering of the age distribution associated with the spatial distribution.

# ACCESS

Australian Community Climate & Earth System Simulator

Collaboration between CSIRO and Bureau of Meteorology Research Centre (to form joint centre) and Australian universities.

- Atmosphere from UKMO (with 4DVAR)
- Ocean (and sea ice) from existing CSIRO/BMRC (based on Princeton)
- Land Surface:
  - Existing CSIRO land surface
  - Add carbon pools based on CASA (with nutrients)
  - Add dynamic vegetation

# Acknowledgments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