347 PROBLEM SET 4, FALL 2006

DUE MONDAY, OCTOBER 2

(1) (20 points) Do Problem (1) of last year's Problem Set 5. This exercise is going to be essential for the rest of this semester. Make sure you understand all the points. Points (c) and (e) refer to the "method of descent", which is a variation of induction we have not introduced yet. Try to look it up (for example in the book) in order to be able to answer this question. You don't need to really understand how it works just yet.

(2) (20 points) For this problem you are allowed to use the fact that the natural numbers are unbounded, i.e. that for every real number r there exists a natural number l such that $l > r$.
Consider the sequence
$$a_n = \frac{n+1}{n}.$$
Prove: For every $\varepsilon \in \mathbb{R}$ with $\varepsilon > 0$, there exists a natural number N such that for any two natural numbers n and m with $n \geq N$ and $m \geq N$ the following holds:
$$|a_n - a_m| < \varepsilon.$$

(3) (20 points) Prove that the set $\mathbb{Q}$ of all rational numbers is countable. You may use everything that was proved in class.

(4) (40 points) Let X and Y be sets, and let $f: X \to Y$ and $g: Y \to X$ be two maps. The map g is called a *right inverse* to f, if (and only if) for every element y of Y the equality
$$y = f(g(y))$$
holds. The map g is called a *left inverse* to f, if (and only if) for every element x of X the equality
$$x = g(f(x))$$
holds. The map g is called an *inverse* of f, if (and only if) it is both a right and left inverse of f.

Date: September 25, 2006.

(In class on Wednesday we will prove that a map f (with nonempty source) is injective if and only if there exists a left inverse of f.)

Prove the following statements:

(a) A map f is surjective if and only if there exists a right inverse of f.

(b) A map f is bijective if and only if there exists an inverse of f.