Tutorial 1 – Algebra 2019

The field with nine elements is obtained from $\mathbb{Z}$ in two steps: first, take the quotient by the equivalence relation generated by $3 \sim 0$, then adjoin the imaginary number i subject to the relation $i \times i = -1$. We will make proper sense of this with a formal definition in class. For now, play around with these rules to fill in the addition and multiplication table for the field with nine elements.

$+$									

$\times$									

Familiarize yourself with the field axioms and check that they hold for the tables you wrote down. Which ones have (consciously or unconciously) influenced your construction? Which is the hardest to check?

There is an element $\alpha \neq 0$ whose powers give all the non-zero elements. Find α and compute its powers.

α	α^2	α^3	α^4				

For each number n from zero to ten, decide whether or not a field with n elements exists. If the answer is "yes", construct this field.