225 PROBLEM SET 5, SPRING 2007

DUE TUESDAY, APRIL 3

(1) (30 points) Let A be the matrix

$$\begin{pmatrix} 8 & 3 \\ 3 & 2 \end{pmatrix}$$

Find an orthogonal matrix U and a diagonal matrix D such that

$$A = UDU^T.$$

(2) (20 points) Carefully draw a picture of the transformation from Question 1. Include and label: both eigenspaces, the unit circle and the ellipse that the unit circle gets mapped to by A. What is length of the longest diameter of the ellipse, what of the shortest one? Use two different colors for both circle and ellipse to illustrate which part maps where.

(3) (40 points) If A is not symmetric, it is still sometimes possible, to diagonalize A, but the base-change matrix will no longer be an othogonal matrix. (We won't get our picture with the ellipse any more, we are stretching along lines which are not orthogonal to each other.) This exercise is supposed to walk you through an example of this. Consider the matrix

$$\begin{pmatrix} 8 & 3 & 4 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) For each eigenvalue, find a corresponding eigenvector.

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(d) Find a diagonal matrix D and an invertible matrix B such that $A = BDB^{-1}$.

(e) Compute BDB^{-1} to check that your answer for the previous question was correct.

(4) (10 points) Think about the following statement and try to explain why it is true: if the characteristic polynomial of a matrix has no roots then this matrix has no eigenvectors.