

GROUP WORK PROBLEMS

- (1) You are given 12 coins and a scale, and you are told that 11 of the coins have the same weight, while one has different weight. You do not know whether the different coin is heavier or lighter than the others. You may use the scales three times in order to find out which of the coins is different. How do you proceed?
- (2) Prove the “only if” part of the following statement: It is possible to draw a connected finite graph G with a single line if and only if the number of odd valued vertices of G is either one or two. Bonus: try to prove the “if” part.
- (3) Prove: Given any 11 real numbers, there are at least two among them whose decimal expansions agree in infinitely many spots.
- (4) Six people are at a party. Some of them shake hands. Prove that there are (at least) three people who mutually shook hands or there are (at least) three people who mutually didn't shake hands (or both).
- (5) Anna and Bernd play a game: each of them writes down a natural number on a sheet of paper, folds it and passes it to the referee. Then the referee writes two numbers on a blackboard that both players can see: one of the numbers is arbitrary, but the other is the sum of the numbers Anna and Bernd chose. The referee asks Anna, whether she can tell him Bernd's number. If she says “no”, he asks Bernd about Anna's number. If he doesn't know it, the referee asks Anna again, and so on. We assume that both Anna and Bernd are intelligent and honest. Prove that after finitely many steps one of them answers “yes”.
- (6) A game starts with two piles of p and q stones. Players A and B take turns starting with A . In each turn a player has to remove one pile and divide the other one into two new piles. The first player who cannot complete a full move loses. For which values of p and q can A force a victory, for which can't he?