347 PROBLEM SET 2, FALL 2005

Look up the definitions of the following objects in the book: *function* (also called "*map*"), *domain* (also called "*source*"), *target, image, graph, well-defined, injective* (also called "*one-to-one*"), *surjective* (also called "*onto*"), *bijective* (also called "*one-to-one correspondence"*).

(1) Consider the map

$$\begin{aligned} f\colon \mathbb{N} &\to \mathbb{N} \\ x &\mapsto 2x. \end{aligned}$$

(Here we used the notation "$x \mapsto 2x$" for "$f(x) = 2x$".)
 (a) What is the image of f?
 (b) Is f surjective?
 (c) Is f injective?

(2) Let X and Y be the sets

$$X = \{a, b, c, d\}$$

$$Y = \{a, b, e\}$$

Draw the graph of each of the following, decide whether it is a well-defined map and if so, whether it is injective, surjective or bijective: (It it possible that several or none of these hold.)

$$\begin{aligned} f\colon X &\to Y \\ a &\mapsto b \\ b &\mapsto e \\ c &\mapsto a \\ d &\mapsto b \end{aligned}$$

$$\begin{aligned} g\colon X &\to Y \\ b &\mapsto e \\ c &\mapsto a \\ d &\mapsto b \end{aligned}$$

Date: September 7, 2005.

$$
\begin{aligned}
h\colon Y &\to X\\
a &\mapsto b\\
b &\mapsto a\\
e &\mapsto a
\end{aligned}
$$

$$
\begin{aligned}
i\colon Y &\to X\\
a &\mapsto b\\
b &\mapsto c\\
a &\mapsto a\\
e &\mapsto d
\end{aligned}
$$

$$
\begin{aligned}
j\colon Y &\to X\\
a &\mapsto b\\
b &\mapsto c\\
e &\mapsto d
\end{aligned}
$$

Try to formulate for a general map f what the properties "well-defined", "injective", "surjective" or "bijective" mean for the graph.

(3) Let now X and Y be arbitrary sets, and let $f\colon X \to Y$ and $g\colon Y \to X$ be two maps. The map g is called a *right inverse* to f, if (and only if) for every element y of Y the equality

$$y = f(g(y))$$

holds. The map g is called a *left inverse* to f, if (and only if) for every element x of X the equality

$$x = g(f(x))$$

holds. The map g is called an *inverse* of f, if (and only if) it is both a right and left inverse of f.

Prove the following statements:

(a) A map f is surjective if and only if there exists a right inverse of f.
(b) A map f is injective if and only if there exists a left inverse for f.
(c) A map f is bijective if and only if there exists an inverse of f.