PROBLEM SET 5, 347, FALL 2005
DUE FRIDAY 10/7

The first question will be what we call "*abstract nonsense*". It is supposed to help you figure out a general principle of how to approach what kind of proof. Here is a list of statements that you might like to prove or disprove:

(1) $\forall x \in X : P(x)$
(2) $\exists x \in X : P(x)$
(3) $P \Rightarrow Q$
(4) P
(5) $\forall n \in \mathbb{N} : R(n)$
(6) $\forall x \in X \mid P(x) : Q(x)$
(7) $\exists n \in \mathbb{N} : R(n)$
(8) $A \subseteq B$
(9) $A = B$
(10) $A \cap B = \{\}$
(11) $\{x \in X \mid Q(x)\} \subseteq \{x \in X \mid P(x)\}$
(12) $\{x \in X \mid P(x)\} \cap \{x \in X \mid Q(x)\} = \emptyset$

Problems:

(1) Each of the following could be the start of a proof for one or several of the above statements, or it could be the start of an argument disproving one of the above, or it could be neither. Indicate which statement it is and whether the proof wants to prove or disprove it.
   (a) Let n be equal to 7. We want to show that $R(7)$ holds.
   (b) Assume that Q is false. We need to show that P is false.
   (c) Let n_0 be the smallest natural number such that $R(n_0)$ is false.
   (d) Let $x \in X$ be such that $P(x)$ is true. We need to prove that $Q(x)$ is false.
   (e) Let n_0 be the smallest natural number such that $R(n_0)$ is true.
   (f) Let $x \in X$ be arbitrary, and assume that $Q(x)$ is true. We need to show $P(x)$.

Date: October 7, 2005.

(g) Fix an arbitrary element $x \in X$, and assume that $P(x)$ is satisfied. We need to prove $Q(x)$.
(h) We are going to show that $R(19)$ is false.
(i) Let $a \in A$ be arbitrary. We need to show that a is not an element of B.
(j) We need to find an $x \in X$ such that $Q(x)$ is true and $P(x)$ is false.
(k) Assume that P is false. We want to lead this assumption to a contradiction.
(l) We need to show "$\subseteq$" and "$\supseteq$".
(m) Let $x \in X$ be arbitrary. We need to show that $P(x)$ is false.
(n) We prove this by induction. Basis step: $R(n)$ is true: ...
(o) Assume that P is false. We need to show that Q is false.
(p) Let $x \in X$ be arbitray but fixed. We need to show that $P(x)$ is true.
(q) We need to find an element y of X such that $P(y)$ is true.
(r) Let a be an arbitrary element of A. We need to show that a is an element of B.
(s) Let $y \in X$ be arbitrary. We need to show $P(y)$.

(2) Do problem 3.41 from the book.

(3) Do problem 3.16 from the book.

(4) Read the definition of "*sequence*" on page 52. Then do problem 3.55 from the book. This is a "*double induction*" argument, which is a variant of induction. Your basis step needs to be that the statement is true for $n = 1$ and $n = 2$. Then in the induction step you are allowed to assume that you already proved the statement for n and for $n-1$ and now want to prove that it is true for $n+1$.