SELECTED SOLUTIONS FOR PROBLEM SET 5, 347, FALL 2005

Here is the list with the correct proof starts.

(1) $\forall x \in X : P(x)$

start of proof: Let $x \in X$ be arbitray but fixed. We need to show that $P(x)$ is true.

start of proof: Let $y \in X$ be arbitrary. We need to show $P(y)$.

(2) $\exists x \in X : P(x)$

start of proof: We need to find an element y of X such that $P(y)$ is true.

start of counter proof: Let $x \in X$ be arbitrary. We need to show that $P(x)$ is false.

(3) $P \Rightarrow Q$

start of proof: Assume that P is true. We need to show that Q is true.

start of proof: Assume that Q is false. We need to show that P is false.

start of counter proof: We need to show that P is true and Q is false.

(4) P

start of proof: Assume that P is false. We want to lead this assumption to a contradiction.

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(5) $\forall n \in \mathbb{N} : R(n)$

start of proof: Let n_0 be the smallest natural number such that $R(n_0)$ is false.

start of proof: We prove this by induction. Basis step: $R(1)$ is true: ... (there was a typo on the problem set)

start of counter proof: We are going to show that $R(19)$ is false.

(6) $\forall x \in X \mid P(x) : Q(x)$

start of proof: Fix an arbitrary element $x \in X$, and assume that $P(x)$ is satisfied. We need to prove $Q(x)$.

(7) $\exists n \in \mathbb{N} : R(n)$

start of proof: Let n be equal to 7. We want to show that $R(7)$ holds.

start of counter proof: Let n_0 be the smallest natural number such that $R(n_0)$ is true.

(8) $A \subseteq B$

start of proof: Let a be an arbitrary element of A. We need to show that a is an element of B.

(9) $A = B$

start of proof: We need to show "$\subseteq$" and "$\supseteq$".

(10) $A \cap B = \{\}$

start of proof: Let $a \in A$ be arbitrary. We need to show that a is not an element of B.

(11) $\{x \in X \mid Q(x)\} \subseteq \{x \in X \mid P(x)\}$

start of proof: Let $x \in X$ be arbitrary, and assume that $Q(x)$ is true. We need to show $P(x)$.

start of counter proof: We need to find an $x \in X$ such that $Q(x)$ is true and $P(x)$ is false.

(12) $\{x \in X \mid P(x)\} \cap \{x \in X \mid Q(x)\} = \emptyset$

start of proof: Let $x \in X$ be such that $P(x)$ is true. We need to prove that $Q(x)$ is false.

(o) proved none of the above, I only included it to confuse you about (3) :)