# 347 PROBLEM SET 3, FALL 2006

Look up the definitions of the following objects in the book or the internet:
*map* (also called "*function*"), *source* (also called "*domain*"), *target, image, graph, (composition of two maps) well-defined, injective* (also called "*one-to-one*"), *surjective* (also called "*onto*"), *bijective* (also called "*one-to-one correspondence*").

(1) **(4 points):** Write down the definitions of *injective* and *surjective.*

(2) **(More negations):**
Complete the following sentences
   (a) **(9 points):** A relation $f \subset X \times Y$ is not a (well-defined) map if and only if ...
   (b) **(4 points):** A map $f$ is not surjective if and only if ...
   (c) **(8 points):** A map $f$ is not injective if and only if ...
   (d) **(3 points):** A map $f$ is not bijective if and only if ...

(3) **(10 points):** Consider the map

$$\begin{aligned} f\colon \mathbb{N} &\rightarrow \mathbb{N} \\ x &\mapsto 2x. \end{aligned}$$

(Here we used the notation "$x \mapsto 2x$" for "$f(x) = 2x$".)
   (a) What is the image of $f$?
   (b) Is $f$ surjective?
   (c) Is $f$ injective?

(4) **(20 points):** Let $X$ and $Y$ be the sets

$$X = \{a, b, c, d\}$$

$$Y = \{a, b, e\}$$

Draw the graph of each of the following, decide whether it is a well-defined map and if so, whether it is injective, surjective or

*Date*: September 8, 2006.

bijective: (It it possible that several or none of these hold.)

$$\begin{aligned} f\colon X &\to Y \\ a &\mapsto b \\ b &\mapsto e \\ c &\mapsto a \\ d &\mapsto b \end{aligned}$$

$$\begin{aligned} g\colon X &\to Y \\ b &\mapsto e \\ c &\mapsto a \\ d &\mapsto b \end{aligned}$$

$$\begin{aligned} h\colon Y &\to X \\ a &\mapsto b \\ b &\mapsto a \\ e &\mapsto a \end{aligned}$$

$$\begin{aligned} i\colon Y &\to X \\ a &\mapsto b \\ b &\mapsto c \\ a &\mapsto a \\ e &\mapsto d \end{aligned}$$

$$\begin{aligned} j\colon Y &\to X \\ a &\mapsto b \\ b &\mapsto c \\ e &\mapsto d \end{aligned}$$

(5) **(12 points):** Try to formulate for a general relation what the property “is a (well-defined) map” means for its graph and for a map $f$ what the properties, “injective”, “surjective” or “bijective” mean for the graph.

(6) **(30 points):** Let $f\colon X \to Y$ and $g\colon Y \to Z$ be injective maps. Prove that this implies that $g \circ f$ is also injective. (You will do the analoguous proof for surjective maps in class on Monday.)