PROBLEM SET 6, FALL 2006
DUE FRIDAY 10/20

You are allowed to use everything you find in the internet or the book or the library or elsewhere, as long as you list your sources and write up the answers to the problems in your own words.

(1) **(10 points)** Prove, using induction, that for any natural number n, we have

$$\sum_{j=1}^{n} j^3 = \left(\sum_{j=1}^{n} j\right)^2.$$

(in case you are not familiar with the notation:) this just means

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = (a + 2 + 3 + \cdots + n)^2.$$

(2) **(10 points)** Prove by induction that for any $q \in \mathbb{R} \setminus \{0\}$, we have

$$\sum_{j=0}^{n} q^j = \frac{1 - q^{n+1}}{1 - q}.$$

This equation is called the *geometric sum equation*. It is proved in the book as Corolloary 3.14, but you should give a direct proof using induction.

(3) **(20 points)** Find a place (for example our book) to read about the Euclidean algorithm. Write down how the algorithm works and do at least two examples which each take at least four algorithm steps. (In this question you only need to say *how* the algorithm works, *not why*.) Make your statement as clear as possible.

(4) **(20 points)** Find a place (for example the book) to read about strong induction. Understand how one can use induction to prove that strong induction works. Then write down the proof in your own words.

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(5) **(40 points)** As an example of strong induction, find a proof for the fact that the Eucledean algorithm works. More precisely, you want to prove that for any given natural numbers a and b, the algorithm ends after finitely many steps and computes the correct thing. (Start by making this statement even more precise, by saying what the "correct thing" is.) I recommend to try strong induction over

$$n := \max\{a, b\}.$$