

**KEY** (made by Yang, Dominique, & Tncia)

100 pts.

① 16 pts.

- 3 pts. a)  $Y=Z$  if and only if  $\forall x \in X : P(x) \Leftrightarrow Q(x)$
- 3 pts. b)  $Y \cap Z = \emptyset$  if and only if  $\forall x \in X : P(x) \Leftrightarrow \neg Q(x)$  or  $\neg(P \cap Q)$
- 6 pts. c)  $Y \cap Z = \{x \in X \mid P(x) \wedge Q(x)\}$   $\neg P \vee \neg Q$   
 $X \setminus Z = \{x \in X \mid \neg Q(x)\}$   $\neg P \vee \neg Q$
- 4 pts. d)  $Y \cup Z = X$  if and only if  $\neg P(x) \Leftrightarrow Q(x)$  } 4 different  
 $\neg P(x) \Rightarrow Q(x)$  } OPTIONS  
 $\neg Q(x) \Rightarrow P(x)$   
 $Q(x) \vee P(x)$

→ (20 pts. TOTAL)

② (4 pts. per answer)

A	B	A $\Rightarrow$ $\neg$ B	$\neg$ B $\Leftrightarrow$ $\neg$ A	$\neg$ B $\Rightarrow$ $\neg$ A	A $\wedge$ $\neg$ B	$\neg$ A $\vee$ B
T	T	F	T	T	F	T
T	F	T	F	F	T	F
F	T	T	F	T	F	T
F	F	T	T	T	F	T

- a) • If A holds then not B is true. / A implies not B  
 • Equivalent to Neither
- b) • B holds if and only if A holds.  
 • Equivalent to Neither
- c) • Not B implies not A.  
 • Equivalent to  $A \Rightarrow B$
- d) • A is true and B is false. / A and not B - equivalent or not = 1 pt.  
 • Equivalent to negation of  $A \Rightarrow B$
- e) • Not A is true or B is true.  
 • Same as  $A \Rightarrow B$ .

(grading:

- table = 2 pts

(each answer  $\frac{1}{2}$  pt.

- statement = 1 pt

$$\neg P \vee \neg Q$$

$$\neg(P \cap Q)$$



For each keys there is at least one blue door that cannot be open

③ (20 points)

A	B	C	a	b	c	d	e	f	g	h	i
T	T	T	T	T	T	T	T	T	T	T	F
T	T	F	T	F	F	T	F	T	T	T	F
T	F	T	T	F	F	T	T	T	T	F	T
T	F	F	T	F	F	F	F	F	T	F	T
F	T	T	T	F	F	F	T	F	T	T	T
F	T	F	F	F	F	F	F	F	F	T	T
F	F	T	F	F	F	F	F	F	F	T	T
F	F	F	F	F	F	F	F	F	F	T	T

$$d=f; c=b; a=g$$

④ (34 pts)

2 pts.

a) Lisa is not old or not happy.

4 pts.

b) There was one day in the last three years, at 3pm, his grandfather was not drinking and was not reading the newspaper.

4 pts.

c)  $\exists x \in X: \exists y \in Y (P(x,y)) : \forall z \in Z: \neg Q(y,z)$

6 pts.

d) All of my little sisters have at least one roommate who has no red bicycle.

2 pts.

e) There exist one heater in Altgeld which works properly.

4 pts.

f)  $\forall y \in Y: \exists x \in X (Q(x,y) : \neg P(x,y))$

4 pts.

g) For every key there exists one blue door in the house which it can not open. -or-

There exists one blue door in the house which cannot be opened by any key.

8 pts.

h)  $\exists \forall l \in L: \forall p \in P \setminus (l,p) \in R \exists ! l' \in L: (l',p) \in R \wedge (\forall q \in P: (l,q) \in R \vee (l',q) \in R)$

negation:  $\exists l \in L: \exists p \in P \setminus (l,p) \in R \exists [ \forall l' \in L: (l',p) \in R \vee ( \exists q \in R: (l,q) \in R \wedge (l',q) \in R ) ] \vee [ \exists l'' \in L: l'' \neq l \wedge l' \in L \wedge l'' \in L: \forall q \in P \setminus (l,q) \in R: (l',q) \in R \wedge (l'',q) \in R ]$

5 (10 pts.)

$$a - \forall \xi \in \Xi : \exists \sigma \in \Sigma \mid \varepsilon(\sigma)$$

$$b - \exists \sigma \in \Sigma : \forall \xi \in \Xi \mid \varepsilon(\sigma)$$

• B is stronger statement.

$$\bullet B \Rightarrow A$$