SELECTED SOLUTIONS FOR PROBLEM SET 3
FALL 2006

Here is the solution for question (6) (only acceptable if written out in whole sentences!)

Let $f : X \to Y$ and $g : Y \to Z$ be injective maps. We need to show that $g \circ f$ is also injective. I.e., we need to show that for any elements $x_1, x_2 \in X$ the implication

$$(g(f((x_1)) = g(f(x_2))) \Longrightarrow (x_1 = x_2)$$

holds.

To prove this, let x_1 and x_2 be arbitrary elements of X and assume that $g(f(x_1)) = g(f(x_2))$. We need to show $x_1 = x_2$. Set $y_1 := f(x_1)$ and $y_2 := f(x_2)$. Then y_1 and y_2 are elements of Y and our assumption states that $g(y_1) = g(y_2)$. Since we know that g is injective, we may conclude that $y_1 = y_2$. Hence we have shown that $f(x_1) = f(x_2)$. Since we know that f is injective, we may deduce that $x_1 = x_2$, as desired.

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