

Algebra

1) For the following rings, determine

whether they are integral domains /
principal ideal domains.

$$\mathbb{Z}, \mathbb{Z}/3\mathbb{Z}, \mathbb{Z}/4\mathbb{Z}, \mathbb{Z}/6\mathbb{Z}, \mathbb{Z}/8\mathbb{Z}, \mathbb{Z}/m\mathbb{Z}$$

$$\mathbb{Z}[x], \mathbb{Z}[x,y], \mathbb{R}[x], \mathbb{R}[x,y], \mathbb{F}_2[x], \mathbb{F}_3[x,y]$$

2) Compute the multiplication and addition

tables of the following rings and determine

whether they are integral domains or fields.

a) $\mathbb{F}_2[x] / \langle x^2 \rangle$

b) $\mathbb{F}_2[x] / \langle x^2 + 1 \rangle$

c) $\mathbb{F}_2[x] / \langle x^2 + x + 1 \rangle$

d) $\mathbb{F}_3[x] / \langle x^2 + 1 \rangle$

e) $\mathbb{F}_3[x] / \langle x^2 + x + 1 \rangle$