Twisted group algebras

Let R be a commutative ring (with 1, as always), and let
$$\beta : G \times G \longrightarrow R^\times$$
be a map. We wish to use β to define a "twisted" version of the group algebra RG.

As R-module, $R^\beta G$ is equal to RG, i.e., it is the free R-module on the set G.

Multiplication on $R^\beta G$ is defined as the linear continuation of
$$g \cdot h := \beta(g,h) \cdot (gh).$$

multiplication in G

multiplication in $R^\beta G$

scalar multiplication.

What conditions does β need to satisfy in order for $R^\beta G$ to be an R-algebra with unit $1 = 1_R \cdot 1_G$?

Interpret $\mathbb{H}$ and $\mathbb{C}$ as such twisted group algebras. Argue that they are not isomorphic to $\mathbb{R}[K_4]$, respectively $\mathbb{R}[\mathbb{Z}/2\mathbb{Z}]$.

We saw last time that a module over $R[G]$ consists of the same data as a group homomorphism
$$G \longrightarrow \mathrm{Aut}_{R\text{-Mod}}(M)$$
for some R-module M. Make an educated guess about modules over $R^\beta[G]$.