

Week 1 Activity Sheet

Groups Categories and Homological Algebra 2018

July 21, 2018

Category theory definitions and examples

For each of the following notions, write out the full definition and then as many examples as you can think of. Work through these examples until you are confident they satisfy the definition

1. Category. Which set-theoretic subtlety did you want to overlook?
2. Functor
3. Natural transformation
4. (a) Product
(b) Coproduct
(c) direct sum

Understand (prove!) the relationship between these three notions.

5. Write down the definitions of
 - (a) Monomorphism
 - (b) Epimorphism
 - (c) Isomorphism
 - (d) Endomorphism
 - (e) Automorphism

Give an example for a morphism that is mono and epi but not iso.

6. (a) Initial Object
- (b) Terminal Object
- (c) Kernel
- (d) Cokernel

In which sense is each of these objects determined by its universal property?

7. (a) Equalizer
 - (b) Coequalizer
8. (a) Limit
 - (b) Colimit

If your category has coproducts and coequalizers of a given cardinality, how can you construct colimits? Prove that your answer indeed satisfies the conditions on a colimit.

9. (a) Additive Category
 - (b) Abelian Category
10. What constructions do you know that yield as result an object determined up to unique (or canonical) isomorphism? Discuss, from a category theoretic point of view, the relevance of this uniqueness.