Tutorial 5 - Algebra 2019

(1) Show that $\mathbb{Z}$ is a PID, UFD, GCD domain, and Euclidean domain. Try to consider the properties you used in order to find a hierarchy between these classes.

(2) Let $\mathcal{R} = \{f : \mathbb{C} \to \mathbb{C} \mid f \text{ is entire}\}$ be the ring of entire functions. Fill out some the following table with Yes or No, explaining each entry.

	GCD Domain	UFD	PID	Euclidean Domain
$\mathbb{Z}[X]$				
$\mathbb{Z}_4$				
$\mathbb{Z}[\mathrm{i}]$				
$\mathbb{R}[X, Y]$				
$\mathcal{R}$				
$\mathbb{Z}[\frac{1}{2}(1+\sqrt{-19})]$				

(3) Use the Euclidean algorithm to find inverses of some elements in $\mathbb{Z}_5[\mathrm{i}]$ and $\mathbb{Z}_2[X]/\langle X^3 + X + 1\rangle$.

(4) Show that 9 is reducible in $\mathbb{Z}[\mathrm{i}]$, and hence show that 3 is not prime (what are the units in $\mathbb{Z}[\mathrm{i}]$? This may be worth proving).

(5) Is $\mathbb{Q}$ a free module over $\mathbb{Z}$?