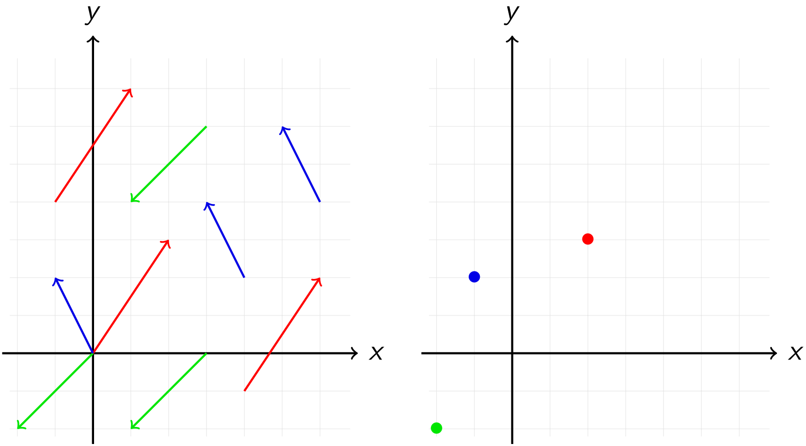
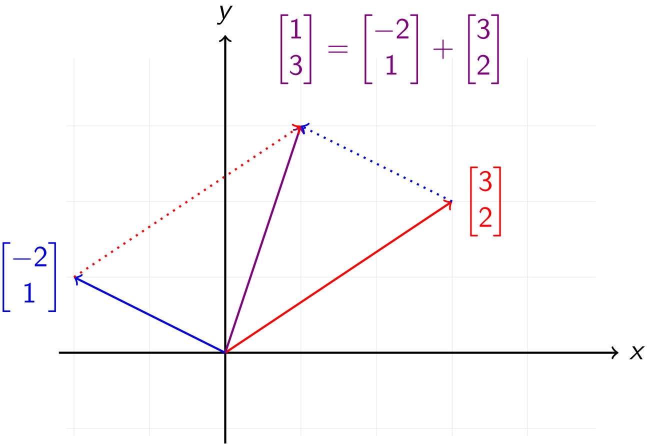
Euclidean vectors and coordinates



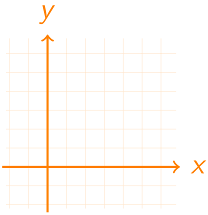
Ways to picture the vectors $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$

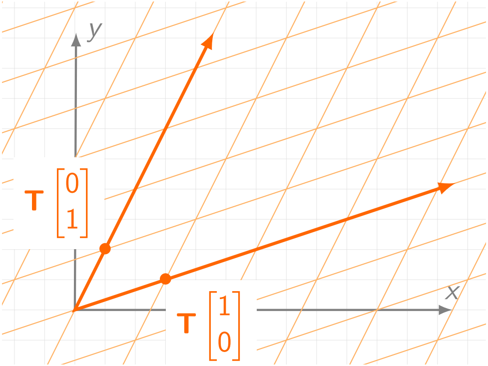
Adding vectors



Arrange your vectors tip to tail!

Linear transformations





send lines to lines and preserve the origin

Algebraically

Since vector addition is described using parallelograms, a transformation T of Euclidean space is linear if and only if

$$T(\vec{u}+\vec{v}) \;=\; T(\vec{u})+T(\vec{v})$$

$$T(a\cdot\vec{v}) \;=\; a\cdot T(\vec{v}).$$

Linear tranformations are exactly those transformations that preserve sums and multiplication by scalars.

This formalism generalises to abstract vector spaces.

The matrix of a linear transformation

In the above picture:

$$\mathbf{T}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

$$\mathbf{T}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

$$\mathbf{T}\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

$$\mathbf{T}\begin{bmatrix}4\\-1\end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

$$\mathbf{T}\begin{bmatrix}x\\y\end{bmatrix} = x\begin{bmatrix} \\ \end{bmatrix} + y\begin{bmatrix} \\ \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$