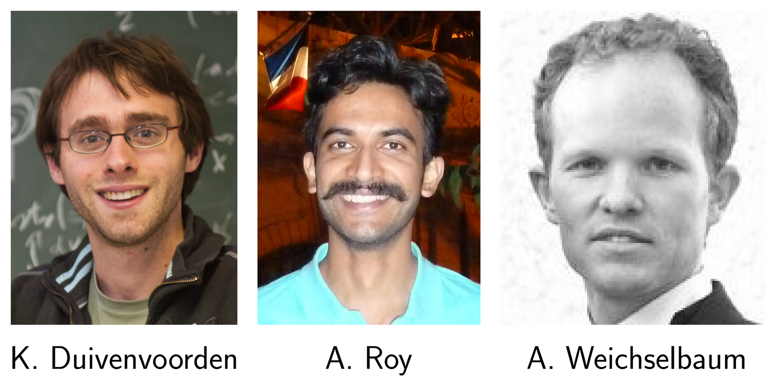
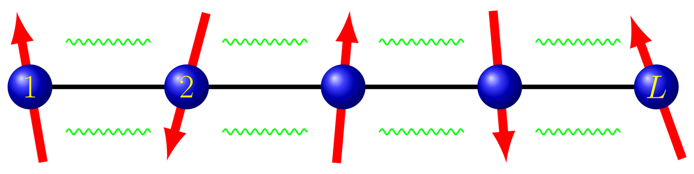
Topological aspects of SU(N) magnetism and its cold atom realization

Thomas Quella (University of Cologne, Institute of Theoretical Physics)



Haldane phases of SU(N) spin chains

Quantum system of spins $\vec{S}_l$ transforming under SU(N)



$\rightarrow$ ground state $|\psi\rangle \in \mathcal{V}^{\otimes L}$

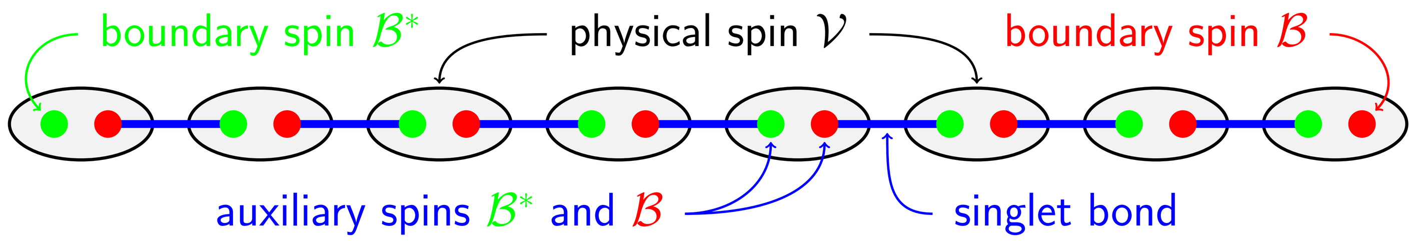
An SU(N)-invariant system is in a **Haldane phase** (see [1]) if

- its ground state $|\psi\rangle$ is unique (no symmetry breaking)
- there is an energy gap (constrains choice of irrep $\mathcal{V}$)
- there is symmetry fractionalization

Fractionalized emergent boundary spins

An open chain may exhibit stable emergent boundary spins

Explicit realization: AKLT-like construction [2]



Symmetry fractionalization:
The center $\mathbb{Z}_N \subset$ SU(N) acts trivially on physical spins $\mathcal{V}$...
...but non-trivially on emergent boundary spins $\mathcal{B}$ and $\mathcal{B}^*$

Classification of Haldane phases

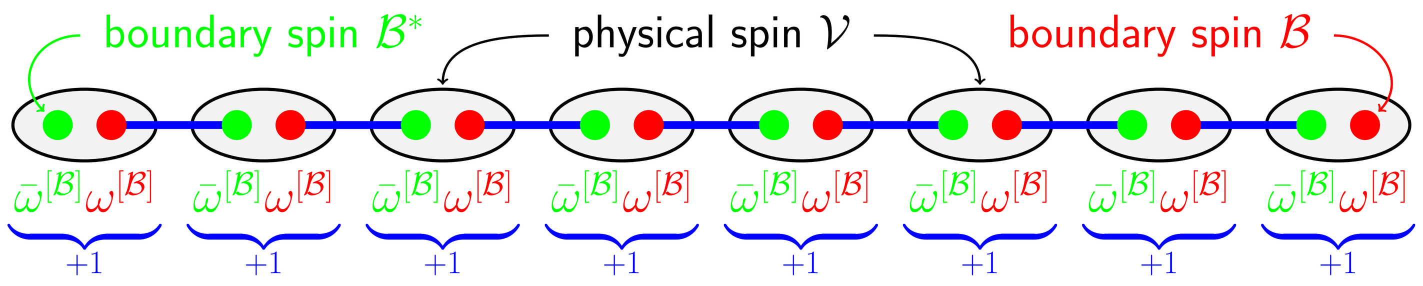
The center of SU(N) is given by

$$\mathbb{Z}_N \cong \{\mathbb{I}, \omega\,\mathbb{I}, \cdots, \omega^{N-1}\mathbb{I}\} \quad \text{with} \quad \omega = e^{\frac{2\pi i}{N}}$$

In an irrep λ, the center acts as $D_\lambda(\omega\mathbb{I}) = \omega^{[\lambda]}\,\mathbb{I}$ where

Young tableau(λ) = ⊞ $\Rightarrow$ $[\lambda] \equiv$ Boxes(λ) mod N

Assuming $[\mathcal{V}] \equiv 0$, the element $\omega\mathbb{I} \in \mathbb{Z}_N$ acts as

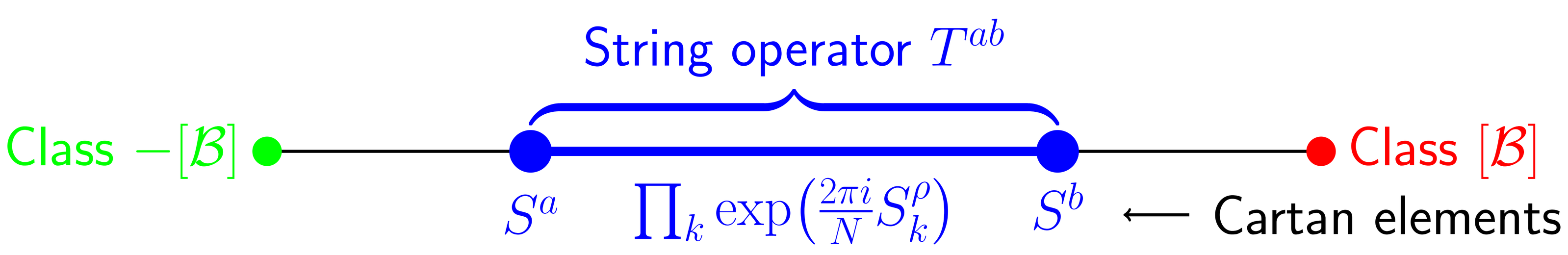


In other words: $\mathcal{V}$ is a linear rep of PSU(N) := SU(N)/$\mathbb{Z}_N$ while the boundary spins $\mathcal{B}$ and $\mathcal{B}^*$ transform in projective reps

Result: PSU(N) admits $N-1$ distinct Haldane phases [3]

Detection in terms of non-local string order

The N phases of PSU(N) spin chains can be distinguished in terms of a matrix-valued string order parameter [4]



In the thermodynamic limit this evaluates to (in a specific basis)

$$\langle T^{ab}\rangle \;\rightarrow\; R\,\exp\left(\frac{2\pi i}{N}(a-b)[\mathcal{B}]\right) \qquad (\text{with } R=0 \text{ for } [\mathcal{B}]=0)$$

Specific realizations

For SU(4) one may, e.g., realize the following Haldane phases

AKLT Hamiltonians for the self-dual rep for $N = 2, 4, 6$ [2, 5, 6]

$$\begin{aligned}
H^{(2)}_{\text{2-site}} &= \tfrac{2}{3} + \vec{S}_1\cdot\vec{S}_2 + \tfrac{1}{3}(\vec{S}_1\cdot\vec{S}_2)^2\\
H^{(4)}_{\text{2-site}} &= \tfrac{8}{3} + \vec{S}_1\cdot\vec{S}_2 + \tfrac{13}{108}(\vec{S}_1\cdot\vec{S}_2)^2 + \tfrac{1}{216}(\vec{S}_1\cdot\vec{S}_2)^3\\
H^{(6)}_{\text{2-site}} &= \tfrac{504}{127} + \vec{S}_1\cdot\vec{S}_2 + \tfrac{47}{508}(\vec{S}_1\cdot\vec{S}_2)^2 + \tfrac{17}{4572}(\vec{S}_1\cdot\vec{S}_2)^3 + \tfrac{1}{18288}(\vec{S}_1\cdot\vec{S}_2)^4
\end{aligned}$$

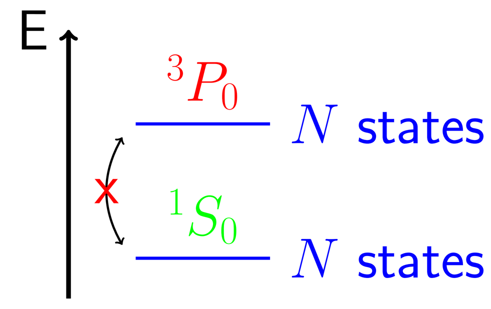
The AKLT Hamiltonians for the adjoint rep are known and involve powers of $\vec{S}_1\cdot\vec{S}_2$ and spin-spin couplings of the form [7]

$$d_{\alpha\beta\gamma}d_{\delta\epsilon\phi}S_1^\alpha S_1^\beta S_1^\delta S_2^\gamma S_2^\epsilon S_2^\phi\ , \qquad d_{\alpha\beta\gamma}\left[S_1^\alpha S_1^\beta S_2^\gamma - S_1^\alpha S_2^\beta S_2^\gamma\right]$$

Notes:
- d is the completely symmetric rank-3 tensor
- The second term breaks inversion symmetry

Cold atom realization

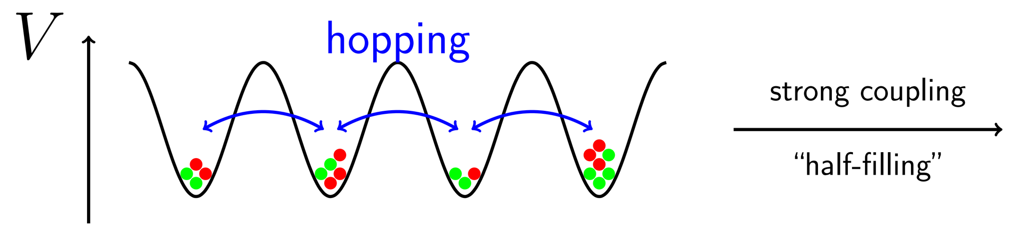
Atom	^{171}Yb	^{173}Yb	^{43}Ca	^{87}Sr
Nuclear spin I	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
$N = 2I+1$	2	6	8	10



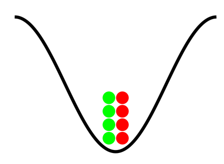
Features of ultra-cold alkaline-earth Fermi gases [8]:

- Large nuclear spin
- Decoupling of electronic and nuclear spin
- Confirmation of SU(N) symmetry in ^{87}Sr and ^{173}Yb [9, 10]

SU(N) Fermi-Hubbard model — SU(N) Heisenberg model



$$H \propto \sum_k \vec{S}_k\cdot\vec{S}_{k+1}$$

Half-filling:  $\Rightarrow$ Physical spin: $\}\ \frac{N}{2}$ (self-dual)

A conjecture on SU(N) Heisenberg spin chains

Claim: The non-trivial inversion-symmetric topological phases above are also realized in the corresponding Heisenberg model

Check: So far only for SU(4) [5, 6, 11]; SU(6) pending [6]
(Adiabatic interpolation, confirmation of edge modes and string order)

Symmetry group	SU(2)	SU(4)	SU(6)	SU(8)	SU(10)	SU(N)
Dim of self-dual rep	3	20	175	1 764	19 404	$\frac{n!(n+1)!}{[(n/2)!(n/2+1)!]^2}$

Literature

[1] F. D. M. Haldane, Phys. Lett. A93 (1983) 464.
[2] I. Affleck, T. Kennedy and E. H. Lieb and H. Tasaki, Phys. Rev. Lett. 59 (1987) 799.
[3] K. Duivenvoorden and T. Quella, Phys. Rev. B87 (2013) 125145 [arXiv:1206.2462].
[4] K. Duivenvoorden and T. Quella, Phys. Rev. B86 (2012) 235142 [arXiv:1208.0697].
[5] H. Nonne, M. Moliner, S. Capponi, P. Lecheminant and K. Totsuka, EPL 102 (2013) 37008 [arXiv:1210.2072].
[6] A. Weichselbaum and T. Quella, *work in progress*
[7] A. Roy and T. Quella, *work in progress*
[8] A. V. Gorshkov et al, Nature Phys. 6 (2010) 289 [arXiv:0905.2610].
[9] X. Zhang et al, Science 345 (2014) 1467 [arXiv:1403.2964].
[10] F. Scazza et al, Nature Phys. 10 (2014) 779 [arXiv:1403.4761].
[11] V. Bois, S. Capponi, P. Lecheminant, M. Moliner and K. Totsuka, Phys. Rev. B91 (2015) 075121 [arXiv:1410.2974].

SFB | TR12
QM² Quantum Matter & Materials
EXCELLENT Funded through the German Excellence Initiative
University of Cologne