TOPOLOGICAL INVARIANTS IN QUANTUM SYSTEMS

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MOTIVATION: HIDING FROM QUANTUM ERRORS

Quantum states are susceptible to two kinds of errors. A classical error flips the qubits whereas a phase error changes the relative signs between them. In order to realise quantum computers, the rates of these errors need to be lowered or eliminated.

A ROBUST NON-LOCALITY: MAJORANA ZERO MODES IN THE KITAEV CHAIN MODEL

The Majorana fermions in the Kitaev chain model are proven to be immune to both types of error. Consider a quantum wire with *L* physical sites sitting on a brick of superconductor. The Hamiltonian of the system is parametrised by three variables: The hopping amplitude ω, the chemical potential μ and the superconducting gap Δ [1].

$$H = -\sum_j \mu\left(a_j^\dagger a_j - \frac{1}{2}\right) + \sum_j \left(-\omega a_j^\dagger a_{j+1} + \Delta a_j a_{j+1} + \mathrm{h.c.}\right)$$

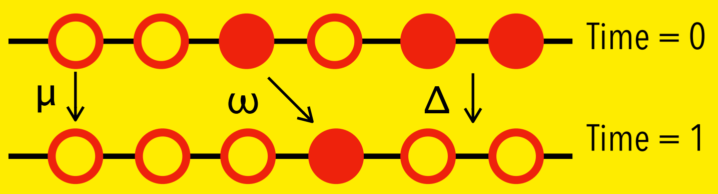

Figure 1: Terms in the Hamiltonian

We associate each physical site on the quantum wire with two Majorana operators defined as

$$c_{2j-1} = a_j + a_j^\dagger, \quad c_{2j} = -i(a_j - a_j^\dagger).$$

These operators are Hermitian as they are their own Hermitian conjugate. They generate a Clifford algebra as they satisfy the anti-commutation relations

$$\{c_i, c_j\} = 2\delta_{ij}.$$

We can rewrite the Hamiltonian in terms of these operators and for some special combinations of parameters, the Majoranas in the ground states are paired up in two different ways. Especially, when $|\Delta| = \omega > 0, \mu = 0$ we end up with two isolated Majoranas in the ground state, which together admit an excitation which is non-local, and immune to errors that are caused by local perturbations.

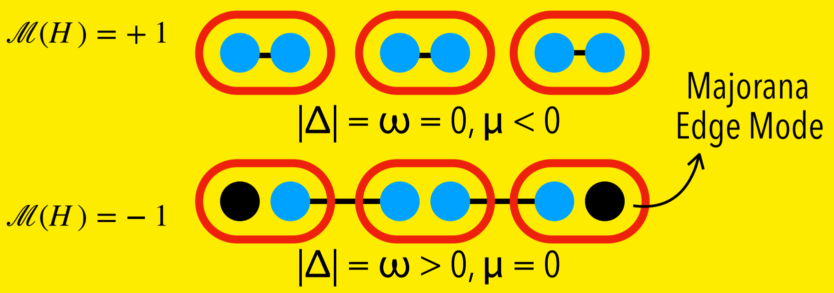

Figure 2: Two ways to pair Majoranas

THE MAJORANA NUMBER: A TOPOLOGICAL INVARIANT

The Majorana edge modes disappear when we close the chain into a loop, and the parity (*1* for Bosonic states and *-1* for Fermionic) of the ground state becomes definite. But the isolated Majoranas in the Hamiltonian are still revealed by the "Majorana number"[1]:

$$\mathcal{M}(H) = \pm 1.$$

We have shown this to be a topological invariant of the Hamiltonian. It is topological in the sense that it remains constant under any continuous deformation of the Hamiltonian provided it does not undergo a phase transition. It is calculated by evaluating the parity of the ground state for two systems, where two chains are closed up in different ways.

$$\mathcal{M}(H) = \frac{P(H(L_1 + L_2))}{P(H(L_1))P(H(L_2))}$$

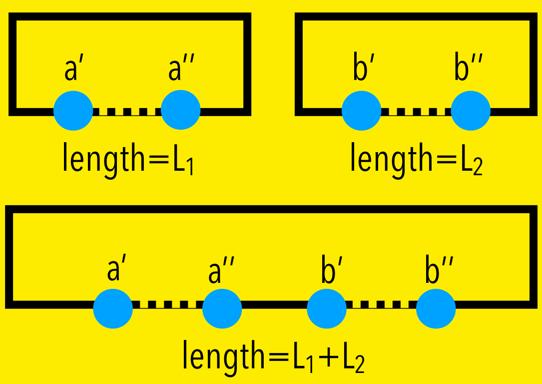

Figure 3: Ways of joining two chains into loop(s)

REFERENCES

[1] Kitaev, A.Y., 2001. Unpaired Majorana fermions in quantum wires. *Physics-Uspekhi*, *44*(10S), p.131.